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**ABSTRACT BOOK**

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# 1. FOREWORD

On behalf of the Organizing Committee, we are very pleased to welcome you to the 2<sup>nd</sup> International Conference of Mathematical Sciences (ICMS 2018) to be held between 31 July 2018 and 6 August 2018 at Maltepe University in Istanbul.

We hope that, ICMS 2018 will be one of the most beneficial scientific events, bringing together mathematicians from all over the world, and demonstrating the vital role that mathematics play in any field of science.

Welcome to our conference, Maltepe University, İstanbul!

**Hüseyin Çakallı**  
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## 2. COMMITTEES

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### 3. SESSIONS

The lectures in the following parallel sessions are to be held after the plenary speakers lectures:

1. **Analytic Topology and Applications**, Prof.Dr. Jiling Cao (Head of Department of Mathematical Sciences, School of Engineering, Computer and Mathematical Sciences, Auckland University of Technology, New Zealand)
2. **Functional Analysis**, Assoc.Prof.Dr. Ayşe Sönmez (Department of Mathematics, Faculty of Science, Gebze Technical University, Turkey)
3. **Mathematical Methods in Physics**, Prof.Dr. Özey Gürtuğ (Maltepe University, Turkey)
4. **Sequences, Series, Summability**, Assist.Prof.Dr. Şebnem Yıldız (Department of Mathematics, Ahi Evran University, Kırşehir, Turkey)
5. **Multidisciplinary Sciences in a New Technology A**, Prof.Dr. Noori F. Al-Mayahi (Head of Al-Khwarizmi Association, Iraq)
6. **Multidisciplinary Sciences in a New Technology B**, Prof.Dr. Noori F. Al-Mayahi (Head of Al-Khwarizmi Association, Iraq)
7. **Other Mathematical Sciences A**, Assist.Prof.Dr. Sibel Ersan (Maltepe University, Istanbul, Turkey)
8. **Other Mathematical Sciences B**, Assist.Prof.Dr. Sibel Ersan (Maltepe University, Istanbul, Turkey)

## 4. ACKNOWLEDGMENTS

We thank firstly the founder of Maltepe University, Hüseyin ŞİMŞEK, the rector of Maltepe University, Prof.Dr. Şahin KARASAR. We also thank the paralell session organizers, and then all scientific committee members who reviewed abstracts which made the conference better.

There are many people who spent a lot of time and effort to make this conference possible. We would like to thank especially to the following young colleagues who had contributed to the success of this conference in various ways:

**Özkan Değer**, İstanbul University, Turkey

**Sibel Ersan**, Maltepe University, Istanbul, Turkey

**Hüsamettin Çapan**, İstanbul University, Turkey

## 5. PLENARY SPEAKERS

The abstracts of the plenary lectures are given in the following.

## Core vs Equilibrium in Economies with Differential Information

Jiling Cao

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A neoclassical exchange economy consists of finitely many agents where each agent has a strictly monotone and strictly convex preference, and a non-zero initial endowment such that the total endowment is strictly positive. Applying analytic topological methods, Arrow and Debreu [1] proved the existence of a competitive equilibrium in such an economy. Furthermore, Scarf [8] proved that the core of a neoclassical exchange economy is non-empty and compact. It is not difficult to observe that every competitive equilibrium is a core allocation, but the converse does not hold in general. In 1964, Aumann [2] established an interesting result showing that the converse holds for an exchange economy with a continuum of agents.

In 1979, Radner [7] introduced the concept of rational expectations equilibrium in economies where traders have different information about the items to be traded. Since then, many mathematicians and economists have been trying to extend the afore-mentioned work of Nobel laureates Arrow, Debreu and Aumann to economies with differential information. Very often, studying such economies needs advanced techniques from modern analysis and topology. In this talk, I will present the recent research work of mine and my coauthors in [3]-[6], as well as others in this direction.

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## Gelfand Theory Unplugged

Robin Harte

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It was Norbert Wiener who observed that whenever a periodic continuous function which never vanishes has an absolutely convergent Fourier series, then so does its reciprocal. Pointwise multiplication generates “convolution” of their coefficient sequences, with a homomorphism from sequences to functions; according to Wiener, if the function is pointwise invertible then also the sequence is “convolution invertible”.

When Israel Gelfand looked at these sequences he saw for the first time what would come to be known as a noncommutative “Banach algebra”. He went on to extend Wiener’s observation from absolutely summable sequences to these Banach algebras, with a completely different and abstract proof. The electricity that powers this “Gelfand theory” is Zorn’s lemma and “maximal ideals”, together with the Gelfand-Mazur lemma, which says that maximal ideals are always generated by bounded multiplicative linear functionals. The “unplugged” version bypasses maximal ideals, and proceeds via the superficially more concrete spectral mapping theorem for finite and infinite systems of Banach algebra elements.

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## Variations of some selection properties

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Selection principles theory is one of the most active areas of topology and its applications in various branches of mathematics. In recent years a number of researches studied a variety of weak versions of the classical selection properties of Menger, Hurewicz and Rothberger. We review some results obtained in the last five-six years, including new results, and discuss some directions of further investigation and open problems in the field.

**Keywords:** Menger-type properties, Hurewicz property, Rothberger property, game theory  
**2010 Mathematics Subject Classification:** 54D20, 91A44

## On the double lacunary $(A, \varphi)$ - statistical convergence of order $\alpha$

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In this paper, our goal is to extend a few results known in the literature for ordinary (single) sequences to multiply sequences of real numbers. Further we introduce an idea of double lacunary strong  $(A, \varphi)$ - convergence of order  $\alpha$  with respect to a modulus function. In addition, we also study some connections between double lacunary strong  $(A, \varphi)$ - convergence of order  $\alpha$  with respect to a modulus and double lacunary statistical convergence of order  $\alpha$

**Keywords:** Modulus function, double Lacunary sequence of order  $\alpha$ ,  $\varphi$ -function, double statistical convergence of order  $\alpha$ .

**2010 Mathematics Subject Classification:** Primary 40H05; Secondary 40C05.

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## 6. ABSTRACTS

The abstracts are ordered by the surname of the presenter of the talk.

This is not a proceedings, the abstracts in this book have been printed as submitted by the authors, after revisions if requested by the referees' and the authors are responsible for the correctness of their abstracts and defending their works during their presentations at the conference.

Selected high quality full length papers presented at the mathematical sessions of the conference can be published in the following journals "Boletim da Sociedade Paranaense de Matemática", "Filomat", "Communications, Series A1: Mathematics and Statistics", "Sakarya University Journal of Science" and "Tamap Journal of Mathematics and Statistics" upon a request of author/ or coauthor after reviewing process.

Extended 4 pages abstracts of high quality papers related to the research areas mainly, Topology, Functional Analysis, Sequences, Series, Summability, Mathematical methods in Physics, will be considered to send to AIP Conference Proceedings and will be evaluated for AIP Conference Proceedings of ICMS 2018. The publication of a 4 page extended abstract will NOT restrict the author(s) from publishing a full-length article on the same topic and with either the same title or with a different title in another journal, provided that this new work is sufficiently enhanced, and cites published extended abstract.



## Mild solutions for semilinear fractional differential equation Via Krasnosel'skii Fixed Point Theorem

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This paper concerns the existence of mild solution for fractional semilinear differential equation with non local conditions. We prove existence by applying the Krasnosel'skii fixed point theorem. An example is given to support our results.

**Keywords:** Krasnoselskii fixed point theorem, Measure of weak noncompactness, Nonlinear integral equation.

**2010 Mathematics Subject Classification:** 47H08, 47H09, 47H10, 47H30.

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## **Interest of the use of the statistical methods of optimization in pharmaceutical development: Application to the formulation of an antiseptic liquid soap**

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In pharmaceutical or cosmetic industry, most products manufactured are obtained by mixing two or several ingredients. This development phase requires many experiments and working time and is often very expensive. In order to develop a product with a good quality and at a lower cost, we often use the experimental research methodology with the objectives of controlling, describing, predicting and explaining the phenomena studied. This methodology has been used to optimize the formulation of an antiseptic liquid soap based on essential oils in accord with specific performance criteria. The experimental strategy who has been adopted consists in optimize, first time, the formulation of a cleansing base composed essentially of three emulsifiers to maximize its viscosity and its transparency and second time, the optimal composition of essential oils to maximize antimicrobial activity of our antiseptic liquid soap. The mixing design's used built according to the SCHEFEE centered experiments matrix required the realization of 29 experiments. The treatment of the results of the empirical modeling of the different points showed satisfactory values of the predictive quality for the two problems posed:  $R^2$  predictive = 0.9419; p-value inferior than 0.0001 for the cleansing base and  $R^2$  predictive = 0.9847; p-value = 0.0016 for the mixing essential oils. Optimization is a branch of mathematics that aims to determine an optimal solution to the posed problems. The mixture designs allowed us to conceive in a short time an economical liquid soap based on essential oils answering the desired quality.

**Keywords:** Optimization method, formulation, mixture design.

## On The joint essential numerical spectrum of operators on Banach spaces

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The purpose of this paper is to introduce the concept of joint essential numerical spectrum  $\sigma_{en}(\cdot)$  of  $q$ -tuple of operators on a Banach space and to study its properties. This notion generalize the notion of the joint essential numerical range.

**Keywords:** Joint numerical spectrum, joint numerical range, joint essential numerical range.  
**2010 Mathematics Subject Classification:** 47A10, 47A12, 47A55.

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## Generalized Inverse Weibull - Generalized Inverse Weibull Distribution

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A new class of continuous distributions based on generalized inverse Weibull has been introduced. As a special case, generalized inverse Weibull - generalized inverse Weibull distribution is proposed. The probability density function, cumulative distribution function, reliability and hazard rate functions are introduced. Furthermore, most important statistical properties of the proposed distribution such as Shannon entropy, relative entropy, stress-strength model have been obtained.

**Keywords:** Generalized Inverse Weibull Distribution, Shannon and Relative Entropies, Stress-Strength Model.

**2010 Mathematics Subject Classification:** Applied Mathematics, Statistics, Mathematical Statistics.

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## Medical Image Watermarking Based Hessenberg Decomposition and DWT

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Medical image watermarking is a mechanism used to embed various types of information to safeguard the privacy of the patient including the patient's name, doctor's digital signature and the view of region of interest for the doctor ... etc into the host image. There are a lot of improved watermark algorithms, however, these information are susceptible to attack when the data are transferred over universal internet channels. This paper proposed a robust watermark algorithm used discrete wavelet transform (DWT) and Hessenberg decomposition. In this algorithm, Hessenberg decomposition applied to each block of the LH band extracted from DTW. In the resulted Hessenberg matrix, the binary watermark bits are embedded in the value which is in situ (1,1) for each block. The experimental results demonstrate that the improvement appears through several metrics, involve PSNR $\approx$ 60 with parameter  $\rho = 2$  (higher robustness against JPEG compression attack) and good imperceptibility against some attacks, to evaluate the fineness of the original with watermarked images and the extracted watermark respectively.

**Keywords:** Hessenberg Decomposition, Discrete Wavelet Transform (DWT), Image Watermark.  
**2010 Mathematics Subject Classification:** 05C50, 65T50, 94A62.

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## Multiple-Point Variogram Features for evolving of Fractal Image Compression technique

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In this work, a new approach to enhance the procedure of fractal image coding is proposed. This method is relied on the new extension of regular variogram function, which called multiple-point variogram. Three textural features were extracted and formed a feature vector to be used as an index of similarity. The objective of this work is to reduce the time consuming that spent in the coding of the remote sensing image. In this approach, the modeled coefficients may not have the appropriate discriminatory effect for identifying different shapes and textures. The issue was overcome by using two functions of modeling variogram; the exponential model and fractal model. This will help in getting better representation of the multiple-point variogram. The result shows a great improvement in the coding process with less time and better resolution. Some efficiency tests are accomplished to prove the effective of the proposed method in comparing to other classical coding methods.

**Keywords:** Fractal dimension, variogram function, remote sensing image, fractal image coding  
**2010 Mathematics Subject Classification:** 28A80,37F05, 81Q35.

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## Steganography Secret Message Using Wavelet 2D Image Fusion

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Steganography is the art of hide security data in any media as pictures, audio, video, text, and protocol, also it can be empathy this secret connection. This research offers Steganography binary secret message of new algorithm to hide message into synthesized image by using Fusion image method to give high security. This system uses three phases in hide secret message: the first phase is used 2D wavelet image fusion between two images in level one (L1), level two (L2), and level three (L3), the second phase is applied db and Haar filters in L1, L2, and L3, and third phase is used Steganography binary secret message by using ASCII to converted each character in secret message in binary number and hide into image, it can be hide each bit in one location in LSB for selection the location by using secret key which rely on the equation:  $2n + 1$  to found position of hide one bit from secret message in synthesized image. The outcome of algorithm is efficient, capacity, transparency, and high security. The system is good in hide secret message stego-image without sensitive for attackers.

Keywords Wavelet 2D Image, AuAgCu alloy nanoparticles, laser modification  
2010 Mathematics Subject Classification: 03E72, 46S40.

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## Constant Curvature of Locally Conformal Almost Cosymplectic Manifold

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**Abstract.** The purpose of the present paper is to study the geometry of locally conformal almost cosymplectic manifold of constant curvature. In particular, the necessary and sufficient conditions in which the locally conformal almost cosymplectic manifold of constant curvature are found. It is proved that such special form of manifolds have nonpositive curvature. The contact analogs of Gray's second and third curvature identities hold on space of constant curvature. The relation between the space of constant curvature and the special classes that depend on conharmonic curvature tensor are found. Finally, the necessary and sufficient condition in which the mentioned manifold is an Einstein manifold are determined.

**Keywords:** Locally conformal almost cosymplectic manifold, conharmonic curvature tensor, constant curvature.

**2010 Mathematics Subject Classification:** 53C55, 53B35.



## On The Weak Compactness Of Weak\* Dunford-Pettis Operators On Banach Lattices

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We characterize Banach lattices on which each positive weak\* Dunford-Pettis operator is weakly (resp., M-weakly, resp., order weakly) compact.

**Keywords:** weak\* Dunford-Pettis operator, weakly compact operator, M-weakly compact operator, order weakly compact operator, DP\* property.

**2010 Mathematics Subject Classification:** Primary 46B42; Secondary 47B60, 47B65.

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## Affinity and well-posedness for a class of optimal control problems

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In this work, we consider a quadratic control problem in finite dimensional space associated with a second order differential inclusion with two boundary conditions. We show that the affinity on the control in the dynamics is necessary for the well posedness of problems without constraints for all desired trajectories.

**Keywords:** Well posedness, quadratic control problem, perturbation, evolution inclusion.

**2010 Mathematics Subject Classification:** 34A60, 28A25, 49K40.

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## Comparison of Standard Deviation, Value at Risk and Expected Loss Deviation Estimation for Single Asset Return in Indonesia Financial Market

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There are two types of risk measures; one based on uncertainty and the other one based on probability of loss. We consider risk measures namely p-norm standard deviation that represent uncertainty and Value at Risk (VaR) represent the probability of worse loss. We also used expected loss deviation (ELD) that represent combination the coherent risk measures and p-norm semi deviation as the protection of penalty from dispersion. Our contribution here is on the empirical study of comparison of those risk measures when it is applied for asset in Indonesia financial market. In the empirical result, we found that the p-norm standard deviation is 1,2935 % (*Rp*.112.28), VaR is *Rp*.158.1624 relative to  $P_{t-1}$  (*Rp*.8625.00) and ELD 0.9%(*Rp*.77.73). ELD indicated that p-norm semi-deviation will represent the losses greater than the expected loss.

**Keywords:** Risk Measures, Standard Deviation, VaR, ELD, Acceptance Set, Semi-deviation, Acceptable Future Net Worth Set.

**2010 Mathematics Subject Classification:** 91B30.

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## Bounded solutions of semilinear time delay hyperbolic equations

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We consider the initial value problem

$$\begin{cases} \frac{d^2 u}{dt^2} + Au(t) = f(u(t), u(t-w)), t > 0, \\ u(t) = \varphi(t), -w \leq t \leq 0 \end{cases}$$

for a nonlinear hyperbolic equation with time delay in a Hilbert space  $H$  with the self adjoint positive definite operator  $A$ . Theorem on the existence and uniqueness of a bounded solution of this problem is established. The application of the main theorem for four different nonlinear partial differential equations with time delay is shown. The first and second order of accuracy difference schemes for the solution of one dimensional nonlinear hyperbolic equation with time delay are presented. Numerical results are given [2].

**Keywords:** Nonlinear delay hyperbolic, bounded solution, numerical results.

**2010 Mathematics Subject Classification:** 35K40, 65M12, 92B05.

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## On the existence of a fixed point of operator equations in Banach algebra

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We present here a new fixed point theorem in Banach algebra for the weak topology in the form of

$$x = Ax + BxCx,$$

$A$ ,  $B$  and  $C$  are operators acting on a Banach algebra with some propriety. Our fixed point results are based on a Kranolsvii fixed point theorem. As an application, we establish an existence results for some type of quadratic integral equations

Keywords: Fixed point theorem, Banach algebra, weak topology,

2010 Mathematics Subject Classification:47H08, 47H09.

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## A Weak Solutions For Some Frictional Contact Problems

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We consider a class of evolutionary variational problems which describe a dynamic process of frictional contact between a piezoelectric body and an obstacle. We derive a variational formulation for the model in a form of a coupled system involving the displacement and the electric potential fields. We provide the existence of unique weak solution of the problems. An example of a dynamic contact with Coulomb's friction is given. The proof is based on the evolutionary variational inequalities, evolution inclusions and Banach's fixed point theorem.

**Keywords:** Evolution inclusion, Evolutionary variational inequality, fixed point, frictional contact, piezoelectric material

**2010 Mathematics Subject Classification:**74M10, 74M15, 47G40, 49J40, 34G25.

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## New Inequalities for Product of Convex Function Classes via New Conformable Fractional Operators

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In this study, some new integral inequalities have been proved for different kinds of convex functions. In the first part, some definitions, basic concepts and fundamental results have been recalled. Second part of our study includes integral inequalities that have been proved via new conformable fractional integral operators for different kinds of convex function classes. In the last section, a short discussion has been achieved on our results.

**Keywords:** Convexity, New Conformable Fractional Operators, Integral inequalities.

**2010 Mathematics Subject Classification:** 26D15.

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## Internal groupoid actions and lifting of crossed modules within the category of groups with operations

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A *groupoid* is a small category in which each arrow has an inverse and a *group-groupoid* is an internal groupoid in the category of groups. By [2] we know that group-groupoids are categorically equivalent to crossed modules. Mucuk and Şahan in [3] have recently defined the notion of lifting for crossed modules and proved that the liftings of a certain crossed module are categorically equivalent to the actions of associated group-groupoid.

The aim of this study is to extend these results to a more general certain category of groups with operations including categories of groups, rings, associative algebras, associative commutative algebras, Lie algebras, Leibniz algebras, alternative algebras and others.

**Keywords:** Internal groupoid, action groupoid, lifting crossed module

**2010 Mathematics Subject Classification:** 18D35, 20L05, 22A05

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## Some results for a certain class of holomorphic functions at the boundary of the unit disk

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**Abstract:** We consider a version of the boundary Schwarz Lemma on a certain class which is denoted by  $K(\alpha)$ . For the function  $f(z) = z + c_2z^2 + c_3z^3 + \dots$  defined in the unit disc  $E$  such that the function  $f(z)$  belongs to the class  $K(\alpha)$ , we estimate from below the modulus of the angular derivative of the function  $\frac{zf'(z)}{f(z)}$  at the boundary point  $b$  with  $\frac{bf'(b)}{f(b)} = \frac{1}{1+\alpha}$ . Moreover, we get Schwarz Lemma for the class  $K(\alpha)$ . We also investigate some inequalities in terms of sharpness.

**Keywords:** Schwarz lemma, Holomorphic function, Jack's lemma, Angular derivative.

**2010 Mathematics Subject Classification:** 30C80, 32A10

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## Modified Mathematical Model of Tumor Treatment by Radiotherapy

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In this research, a mathematical model of tumor treatment by radiotherapy is studied and a new modification for the model is proposed as well as introducing the check for the suggested modification. Also the stability of the modified model is analyzed in the last section.

**Keywords:** Cancer, Dynamical systems, radiotherapy.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## Bi-Domination in Graphs

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In this paper, new definition of domination in graphs is introduced called bi-domination. Let  $G = (V, E)$  be a graph. A set  $D$  which is a proper subset of  $V(G)$  is a bi-dominating set of  $G$  if for every  $v$  in the set  $D$  dominates exactly two vertices in set  $V - D$ . The minimum cardinality of a bi-dominating set of  $G$  is called a bi-domination number of  $G$  and is denoted by  $b^i(G)$ . Some theorems, propositions and observation of bi-domination graph are determined.

**Keywords:** Dominating set, bi-domination number, bi-dominating set.  
**2010 Mathematics Subject Classification:** 05C69, 81Q30, 97K30.

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## Synthesis of Au-Ag-Cu Trimetallic Alloy Nanoparticles Prepared by Electrical Exploding Wire Technique Combined with Laser Pulses Shooting in Distilled Water

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Formation of Au-Ag-Cu ternary alloy nanoparticles (NPs) is of particular interest because this trimetallic system have miscible (Au-Ag and Au-Cu) and immiscible (Ag-Cu) system. So there is a possibility of phase segregation in this ternary system. At this challenge it was present attempts synthetic technique to generate such trimetallic alloy nanoparticles by exploding wire technique. The importance of preparing nanoparticles alloys in distilled water and in this technique makes the possibility of obtaining nanoparticles free of any additional chemical substance and makes it possible to be used in the treatment of cancer or diseases resulting from bacterial or virus with least toxic. In this work, three metals alloys Au-Ag-Cu nanoparticles(A,B, and C) were prepared by exploding wire with different ratio of each elements (A: 50 percent Au, 25 percent Ag, 25 percent Cu, B: 25 percent Au, 50 percent Ag, 25 percent Cu, C: 15 percent Au, 60 percent Ag, 25 percent Cu). A high purity wire with diameters (0.3mm) against plate of these alloys were held at 20V with respect to the wire achieving different currents of 75, 100 and 160 A in distilled water and then the size and a shape of the synthesized alloy nanoparticles modify by pulse laser with different energies (400 ,600 and 800 mJ. ), where the colloids of nanoparticles were exposed to one thousand pulses of 532 nm wavelengths per pulse from second harmonic Nd-YAG laser, after it has been focused by a lens with 15 cm focal length.

Keywords Exploding Wire, Au Ag Cu alloy nanoparticles, laser modification  
2010 Mathematics Subject Classification: 03E72, 46S40.

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## Soft Ideal of Soft Banach Algebra ( $S.B.A$ )

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The material in this paper is mainly related to the soft Banach algebra. The aim of this paper is to provide basic information about a soft ideal of soft Banach algebra. Another types of soft ideal (soft proper ideal, soft maximal ideal) in ( $S.B.A$ ), these concepts are illustrated by some results.

**Keywords:** Soft Banach algebra, Soft ideal, Soft proper ideal, Soft maximal ideal.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## On Idea of Soft Gem-Set In Soft Topological Space

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In this research, we introduce idea of Soft Gem-Set in soft topological space and study the most important characteristics for this concept and join it with topological concepts.

**Keywords** soft metric space, fuzzy soft metric space, coupled fixed point theorems.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## A New Modified Playfair Cipher

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The methods of encryption and decryption are used to protect the transferred information through public channels. The symmetric cipher methods that are based on the shared secrecy are preferable for this purpose. Playfair cipher is one of the classical symmetric ciphers; it has some limitation because it uses 5x5 matrix. To overcome this limitation, we propose in this paper a new encryption and decryption method based on the classical Playfair cipher by expanding it to  $nn$  matrix with Permutation and transposition. The Security and the efficiency of the proposed method are demonstrated in comparing to the traditional and modified Playfair ciphers.

**Keywords:** Playfair, symmetric ciphers, classical cryptography, information theory  
**2010 Mathematics Subject Classification:** 11T71,14G50, 94A60.

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## Optimal Simultaneous Controller for Finite Collection of Linearized Robotic System by Non-classical Variational approach

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The robotic dynamic system for one leg of walking two links have been considered. The state-space models of finite collection of linearized robotic system on some equilibrium points with help of implicit function theorem and Newton-Raphson method have been derived. Some optimal performance indices have also been defined to minimize the velocity, position and applied torque with energy consumption. The necessary and sufficient conditions for simultaneous linear-quadratic models are derived, to obtain a finite collection of two-points boundary value problems with a suitable initial and boundary conditions. The non-classical variational formulation mixed with direct Ritz-method have been developed for the solvability point of view. The mathematical justification for the solvability and uniqueness of the obtained finite-collection two-boundary value problems using this approach have been discussed and proved..

**Keywords:** Robot, simultaneous of optimal control, non-classical variational approach .

**2010 Mathematics Subject Classification:** 70E60, 49N05,70B15

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## Using Two Local Search Methods to Finding Approximate Solutions of The Sum of Two Criteria

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In this paper, we considered the problem of scheduling  $n$  jobs on single machine to find approximate solutions for the sum of cost of total flow time and maximum late work, when the jobs may have unequal release dates, this problem is strongly NP-hard. We present two local search methods to find Artificial Fish Swarm Algorithm (AFSA) and Fruit Fly Optimization Algorithm (FOA). The problem had been solved with up to 6000 jobs.

Keywords: Flow time, , maximum late work, Scheduling.

2010 Mathematics Subject Classification: 03E72, 46S40.

## Experiments with Terai's Conjecture

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Let  $a, b, c$  be relatively prime fixed positive integers greater than one. Although the exponential Diophantine equation  $a^x + b^y = c^z$  in positive integers  $x, y, z$  has been studied by a number of authors extensively from various point of view, there are some conjectures still remain unsolved in general. One of them is Terai's Conjecture which states that if  $a, b, c, p, q, r$  are fixed positive integers satisfying  $a^p + b^q = c^r$  with  $a, b, c, p, q, r \geq 2$  and  $\gcd(a, b) = 1$ , then the above equation has only the positive integer solution  $(x, y, z) = (p, q, r)$  except for a handful of triples  $(a, b, c)$  [1, 2]. Exceptional cases are listed in [3]. Although this conjecture is proved to be true in many special cases, it is still remaining an unsolved problem yet. In this study we examine a specific case of the above equation in which the Terai's Conjecture is true.

**Keywords:** exponential Diophantine equations, integer solution, Terai conjecture.

**2010 Mathematics Subject Classification:** 11D41, 11D61, 11D75.

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## Boundary Behaviour of Holomorphic Functions on the Cardioid Domain with some Applications

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The objective of this research paper is to show how the *Brennan's* conjecture becomes a useful tool to construct a holomorphic function on the cardioid domain  $\Omega$ , where  $\phi'(0) = 0$ , for  $0 < n \leq 1$ ,  $n \in \mathbb{N}$  and another belongs to *Hardy space*  $H^{\frac{2n\pi-\theta}{n\pi}}(\mathbb{D})$ ,  $n \in \mathbb{N}$ , on the boundary of unit disk. Moreover, we have addressed some applications on the existence of cusp on the boundary of arising from integrability of conformal maps through one of the polar functions in the general solution of *Laplace* equation.

**Keywords:** *Brennan's* conjecture, Conformal mapping, *Cardioid* domain, and *Laplace* equation.  
**2010 Mathematics Subject Classification:** 30-XX.

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## Mathematical Model of Airflow in A Bifurcated Trachea

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An accurate determination of the airflow pattern inside a bifurcated trachea is a challenging task in breathing physiology. Understanding of flow behaviour in human airways is a prerequisite for the treatment of breathing patients. A new mathematical model for simulating airflow inside human trachea and main bronchi under resting and normal breathing scenario on the unsteady flow is determined. The governing equations of motion, consisting of unsteady, nonlinear, non-homogeneous, Navier-Stokes equations are derived and numerically solved using the Marker and Cell method. A two-dimensional cylindrical coordinate system with appropriate initial and boundary conditions are used. The discretization is performed on uniform staggered grids. The pressure is calculated iteratively using the Successive-Over-Relaxation method. Quantities, including the pressure loss, axial velocity, volumetric flow rate, flow resistance and streamlines of airflow patterns are computed. The computed axial velocities are agreed when compared with other experimental and numerical findings. An increase in the Reynolds numbers is found to enhance the pressure loss inside the trachea. Thus, it generated a higher negative pressure in the lungs. Simulation results are demonstrated to be accurate when compared with the real situation.

**Keywords:** Numerical study , pressure correction, bifurcated trachea.

**2010 Mathematics Subject Classification:** 03E72, 46S40. **References**

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## On the Periodicity of the Solution of a Rational Difference Equation

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In this paper, some cases on the periodicity of the rational difference equation

$$S_{n+1} = S_{n-p} \left( \frac{aS_{n-q} + bS_{n-r} + cS_{n-s}}{dS_{n-q} + eS_{n-r} + fS_{n-s}} \right), \quad (1)$$

are investigated, where  $a, b, c, d, e, f \in (0, \infty)$ . The initial conditions  $S_{-p}, S_{-p+1}, \dots, S_{-q}, S_{-q+1}, \dots, S_{-r}, S_{-r+1}, \dots, S_{-s}, \dots, S_{-s+1}, \dots, S_{-1}$  and  $S_0$  are arbitrary positive real numbers such that  $p > q > r > s \geq 0$ . Some numerical examples are provided to illustrate the theoretical discussion. Our main theorem has the following form.

**Theorem** Equation (1) has no positive solutions of prime period two  $\forall a, b, c, d, e, f \in (0, \infty)$  in all the following cases:

- 1) The positive integers  $p, r, s$  are even and the positive integer  $q$  is odd.
- 2) The positive integers  $p, q, r$  and  $s$  are even.
- 3) The positive integer  $p, q, r$  are even and the positive integer  $s$  is odd.

**Keywords:** Rational Difference equations, Periodicity, Higher-Order.

**2010 Mathematics Subject Classification:** 39A10.

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## Prediction of OPEC oil prices using different versions of(GARCH) models in time series

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Many time series are characterized by their large volatility over time, especially time series related to the movement of the economy, And those that relate to the change in stock prices or the movement of financial transactions and stock markets characterized by such phenomena of instability over time, which makes it the problem of heterogeneity of disparities clearly. Since the time series analysis is required to construct the appropriate model for forecasting the future of the studied phenomenon is to achieve stability on average and consistency of homogeneity of variance, The aim of this study was to study the use of predictive models that can adapt to time series with large fluctuations over time, A number of important models used to deal with heterogeneous time series in variance, GARCH, GARCH-M, and EGARCH have been studied and reviewed when the distribution of errors follows normal distribution, , Which was first used by the researcher Engle in 1982 and developed by other researchers, The characteristics of these models have been reviewed and applied for the purpose of forecasting the daily prices of oil according to the prices approved by OPEC for the period from 2003 to 2018, The study and practical analysis of oil price data showed that the best model for forecasting fluctuations in oil prices for the period mentioned is the EGARCH (2,1), By adopting some important criteria for selecting the best model such as AIC, SIC, H-QIC.

Keywords EGARCH,homogeneity, H-QIC.

2010 Mathematics Subject Classification: 03E72, 46S40.

## **Serum Levels of (lead, cadmium and silver) in Patients with Breast Cancer compering in health females in Iraq**

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Trace elements are major components of biological structures; however, excessive levels of these elements can be toxic. In the present study, serum levels of trace elements were measured in 25 patients with newly diagnosed as breast cancer and 25 healthy volunteer by using atomic absorption spectrophotometry. It was found that there was no statistically significant in serum levels of Ag, Pb and Cd when compare the mean of concentration of these trace elements for patients with health ( $0.05 > p$ ), These changes may be important in the pathogenesis of breast cancers; however, further prospective studies are needed to identify relationships between breast cancer and trace elements.

**Keywords:** breast cancer , trace element , serum levels.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## The Energy of Tripartite Graphs

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The energy of a graph is used to approximate the total  $\pi$ -electron energy of molecules. Thus, finding a new technique that facilitates this task is a challenge that has received increased research attention. This paper introduces the eigenvalues of a complete tripartite graph  $T_{i,i,n-2i}$ , for  $n \geq 4$ , with respect to the adjacency, Laplacian, and signless Laplacian matrices. The extreme eigenvalues of the energy of irregular graphs in terms of its degree sequence is found to characterize the component with the maximum energy. The chemical HMO approach is particularly successful in the case of the total  $\pi$ -electron energy. Experiments reveal that some chemical components are equienergetic with the tripartite graph. This finding helps easily derive the HMO of most of these components despite their different structures.

**Keywords:** Tripartite graph, Adjacency matrix, Laplacian matrix, Signless matrix, Characteristic polynomial.

**2010 Mathematics Subject Classification:** 68R10, 81Q30, 97K30.

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## Numerical solutions for a class of bilevel problems

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Bilevel optimization problems can be defined as optimization problems in which the definition of the feasible set passes by the resolution of another optimization problem called parametric problem. These problems are considered as sequential games that have their origin in the game theory of Stackelberg. In order to clarify the hierarchy between the two players, the first decision maker is called Leader and the second is called Follower. From a mathematical point of view, a bilevel optimization problem consists of two problems included in a single instance. The control of the variables is partitioned among the decision makers who try to optimize their own objective functions. The Leader makes the first decision in order to optimize its objective function. The Follower observes the Leader's decision and reacts rationally [1]. In this work, we consider a nonlinear bilevel programming problem. By reformulating the problem into a standard mathematical program (with one level), we examine a nonconvex approach based on DC optimization [2] for its resolution. The proposed approach is first tested on bilevel problems whose solutions are known and then an experimental study on randomly generated problems is conducted to evaluate its performance.

**Keywords:** Bilevel programming, Stackelberg game, DC optimization.  
**2010 Mathematics Subject Classification:** 47N10, 90C25.

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## Compact finite difference scheme for timoshenko beam equation

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This paper deals with the numerical approximation problem of the optimal control problem governed by the Timoshenko beam equation with locally distributed feedback. We apply a compact finite difference approximation of fourth-order for the discretizing spatial derivatives of these equations and the Crank-Nicolson method for the resulting linear system of ordinary differential equations. The proposed method has fourth-order accuracy in space and two-order in time variables, i.e. this method is of order  $O(h^4, k^2)$ . Additional to high-order of accuracy, the proposed method is unconditionally stable which will be proved in this paper. Numerical results show that the compact finite difference approximation of fourth order and the Crank-Nicolson method give an efficient method for solving the Timoshenko beam equations.

**Keywords:** Compact finite difference, Stability.

**2010 Mathematics Subject Classification:** 65M06, 65N12.

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## Well-posedness of nonlocal elliptic differential and difference problems

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The nonlocal boundary value problem for the elliptic differential equation

$$-v''(t) + Av(t) = f(t) \quad (0 \leq t \leq T), v(0) = v(T) + \varphi, \int_0^T v(s)ds = \psi$$

in an arbitrary Banach space  $E$  with the positive operator  $A$  is studied. The well-posedness of this problem in various Banach spaces is established.

The second order of approximation two-step difference scheme is presented. The well-posedness of this difference scheme in various Banach spaces is established. In applications, the coercive stability estimates in Holder norms for the solutions of three type elliptic differential and difference nonlocal problems are obtained. Additionally, some illustrative numerical results are provided.

**Keywords:** Well-posedness, positive operators, elliptic equation.

**2010 Mathematics Subject Classification:** 35J25, 47E05, 34B27.

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## Source identification hyperbolic differential and difference problems

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The source identification problem for a one dimensional hyperbolic equation is studied. Operator approach of [3] permitted us to establish stability estimates for the solution of the source identification problem. The first and second order of approximation difference schemes for the numerical solution of source identification problem are investigated. Numerical results are presented.

**Keywords:** Source identification problem, hyperbolic differential equations, difference schemes.

**2010 Mathematics Subject Classification:** 35J25, 47E05, 34B27.

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## Some Types of Normality with Respect to Ideal

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The main focus of our paper is to introduce  $\kappa\mathcal{I}$ -normal space. Characterizations and properties of the space are studied and investigated. Moreover, preservation under mappings and subspaces are examined.

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Keywords:  $\kappa\mathcal{I}$ -normal,  $\mathcal{I}$ -normal, ideal topological space.

2018 Mathematics Subject Classification: 54D15, 54C10.

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## Algebraic Invariants and Bifurcation Analysis of a Nonlinear Financial System by Computational Algebra Methods

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This paper is concerned with the algebraic invariants and bifurcation analysis of a complicated nonlinear financial system studied by MA Jun-hai et. al. [1, 2]. We focus on the the stability of the equilibrium points of the system, parameter conditions for all codimension-1 bifurcations such as Hopf bifurcation, and limit cycles arising as a consequence of Hopf bifurcations.

**Keywords:** Bifurcations of limit cycles, Stability, Computational Algebra.  
**2010 Mathematics Subject Classification:** 37G15,34D20, 37M20.

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## Division Operation of Number Learning through Picture Lapping Conducted to Students with Light Mental retardation in Palembang

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This qualitative descriptive research aims at describing the operation of number learning through picture clapping conducted to students with light mental retardation at Institute for physically and mentally handicapped children in Palembang. This study involves four students at the eighth grade as the subjects. Video recording and observation are undertaken to collect the data. The students' learning process will be recorded in video and later compared to the results of the observation. During the learning process using the selected strategy, teachers' role is as the facilitator to help the pupils translating the repeated minus question to be the division operation. Thus, in the end of the class, students are able to do the exercises given.

**Keywords:** clapping picture, the repeated minus question; division operation; light mental retardation student.

**2010 Mathematics Subject Classification:** 97U30

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## On 1-absorbing primary ideals of commutative rings

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Let  $R$  be a commutative ring with nonzero identity. In this paper, we introduce the concept of 1-absorbing primary ideals in commutative rings. A proper ideal  $I$  of  $R$  is called a *1-absorbing primary* ideal of  $R$  if whenever nonunit elements  $a, b, c \in R$  and  $abc \in I$ , then  $ab \in I$  or  $c \in \sqrt{I}$ . Some properties of 1-absorbing primary ideals are investigated. For example, we show that if  $R$  admits a 1-absorbing primary ideal that is not a primary ideal, then  $R$  is a quasilocal ring. We give an example of a 1-absorbing primary ideal of  $R$  that is not a primary ideal of  $R$ . We show that if  $R$  is a Noetherian domain, then  $R$  is a Dedekind domain if and only if every nonzero proper 1-absorbing primary ideal of  $R$  is of the form  $P^n$  for some nonzero prime ideal  $P$  of  $R$  and a positive integer  $n \geq 1$ . We show that a proper ideal  $I$  of  $R$  is a 1-absorbing primary ideal of  $R$  if and only if whenever  $I_1 I_2 I_3 \subseteq I$  for some proper ideals  $I_1, I_2, I_3$  of  $R$ , then  $I_1 I_2 \subseteq I$  or  $I_3 \subseteq \sqrt{I}$ .

**Keywords:** Prime ideal, primary ideal, 1-absorbing primary ideal, 2-absorbing primary ideal, 2-absorbing ideal, weakly prime ideal, weakly primary ideal, weakly 2-absorbing primary ideal, weakly semiprime ideal,  $n$ -absorbing ideal.

**2010 Mathematics Subject Classification:** Primary 13A15, 13F05; Secondary 05A15, 13G05.

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## Uniform boundedness and extinction results of solutions to a predator-prey system

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Uniform boundedness and extinction results of solutions to a system of reaction-diffusion equations on unbounded domains modeling two species on a predator-prey relationship is considered.

**Keywords:** Reaction-diffusion, semigroups, local existence, global existence, positivity, predator-prey, extinction of solutions.

**2010 Mathematics Subject Classification:** 35A01, 35A02, 35B09, 35B50, 35k57.

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## Image compression using Hilbert scan and B-spline curve approximation

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In this paper, effectiveness of B-spline curves has been studied for compression of grayscale images. The two dimensional digital image is converted into one dimensional grayscale sequences by using Hilbert scan. The B-spline curves are used to approximate the scanning data points, and the approximate parameters are stored to code the corresponding data points. As a result, the proposed algorithm has lower BR and better PSNR than the scheme based on quadratic Bezier curve approximation.

Keywords: Image compression, Hilbert scan, B-spline curve, Least squares curve approximation.  
2010 Mathematics Subject Classification: 65D17, 94A08, 68U10.

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## **The effect of quenching temperature on the mechanical and thermomechanical behavior of poly (methylmethacrylate)**

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In this study, the effects of free quenching on mechanical and thermomechanical behaviors of poly (methylmethacrylate) (PMMA) have been investigated. Two different thermal treatments were used: a quenching from  $T_g + 15C$  and an annealing. The results have shown that an improvement of the Vickers hardness can be obtained after a quenching at  $20C$ . The improvement of Vickers hardness is linked to the relaxation mode of the material located around this temperature.

Keywords: PMMA, mechanical properties, Quenching temperature, thermomechanical properties.  
2010 Mathematics Subject Classification: 82D60, 82D30 ,74F05.

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## Gaussian wave packet for general time-dependent generalized harmonic oscillator

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The Gaussian wave packet (GWP) has been extensively by physicist and chemist in studying the time evolution of various dynamical systems. Gaussian wave packets are appearing in quantum mechanics and are fundamental states of many physical systems that exhibit various nonclassical properties. We obtained the exact solution of the Schrödinger equation for a general time-dependent generalized harmonic oscillator, which is the semi classical Gaussian wave packet, centred on the classical guiding trajectory  $(p, q)$ .

**Keywords:** Schrödinger equation; Gaussian wave packets; time-dependent system.  
**2010 Mathematics Subject Classification:**58-XX.

## The exact sequence of Whitehead

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Let  $X = (X^k)_{k \leq 2}$  be a simply connected CW complex. The Whitehead exact sequence of  $X$  [1], denoted by  $W.E.S(X)$ , is defined by

$$\cdots \longrightarrow H_{k+1}(X) \longrightarrow \Gamma_k(X) \longrightarrow \pi_k(X) \longrightarrow H_k(X) \longrightarrow \cdots$$

where  $h_k$  is the Hurewicz homomorphism and where

$$h_k(X) = \ker \pi_k(X^k) \longrightarrow \pi_k(X^k, X^{k-1})$$

Given two simply connected CW complexes  $X$  and  $Y$ . In this talk we define the notion of coherent homomorphism  $\theta$  from  $W.E.S(X)$  towards  $W.E.S(Y)$  and we prove the following theorem

**Theorem 1.** Two simply connected CW-complexes  $X$  and  $Y$  are homotopic if and only if  $\theta$  is a coherent isomorphism.

Keywords: Homotopy types, Whitehead's exact sequence, .

2010 Mathematics Subject Classification: 55P10,

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## On a system of Coupled Differential Equations with Nonlocal integral Conditions

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**Abstract.** We are interested in finding a solution to a system of coupled differential equations subject to nonlocal types integral conditions. We reduced the above mentioned problem to an abstract Cauchy problem. The existence of a solution is proved using the theory of semigroups with singularities.

**Keywords:** System of differential equations, integral conditions, semigroups with singularities.  
**2010 Mathematics Subject Classification:** 65J22, 47D06, 34G10.

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## Singulaire dynamic method, and singular perturbation problem

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This work is devoted to singular transport phenomena by convection with fast-low time scales, or to transport in porous media with vanishing discontinuous porosities. For  $P1 - P0$  finite element, by using a reduction of the approximation order for the time differential operator, we propose a numerical method which does not have any oscillations in the neighborhood of the coefficient discontinuity. Error estimates of order one with respect to space are provided. Euler explicit and implicit time schemes are proposed, and by considering a toy problem, the order one of convergence with respect to time and space is checked.

**Keywords:** Singulaire dynamic method , hyperbolic operators, matrix of singular mass  
**2010 Mathematics Subject Classification:** 65M15, 65M60, 35F16.

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## On certain means generated by generalized trigonometric functions

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In 1995, P. Lindqvist [4] studied generalized trigonometric functions depending on a parameter  $p > 1$  which for the case  $p = 2$  reduce to the familiar functions. As an application of generalized trigonometric and hyperbolic functions [1, 2, 4], here author generalize logarithmic mean  $L$ , Neuman-Sándor  $M$ , two Seiffert means  $P$  and  $T$ , see[5]. Moreover, several two-sided inequalities involving these generalized means are established.

Keywords: Hypergeometric functions, classical mean, two Seiffert means.  
2010 Mathematics Subject Classification: 34L10, 33E05, 33C75.

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## Existence of Periodic Solutions For Delay Integro-dynamic Equations On Time Scales

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In this work we use Schauder's fixed point theorem to show the existence of positive periodic solutions for an integro-dynamic equations on time scales. An examples is given to illustrate this work.

**Keywords:** Integro-dynamic equations, Schauder fixed point theorem, periodic solution, time scales  
**2010 Mathematics Subject Classification:** 34K13, 34A, 34K30, 34L3.

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## Approximations of arithmetic functions

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In this talk, we provide some approximations of arithmetic functions, and we present classical and nonclassical notes on the solvability of certain Diophantine inequalities involving multiplicative functions such as  $\varphi$ ,  $\tau$  and  $\psi$ . In the framework of internal set theory we use the positive integers  $n$  whose prime factors are unlimited and its number of distinct prime factors is limited.

Keywords: arithmetic function, Diophantine inequality, prime numbers, internal set theory..  
2010 Mathematics Subject Classification: 03H05, 11A05, 11A25.

## On some retarded integral inequalities in two variables and applications

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Integral inequalities which were introduced by GronwallBellman [1] and their various generalizations play a fundamental role in the study of qualitative properties of solutions of differential equations, integral equations and integral differential equations. Recently, many retarded versions of GronwallBellmanPachpatte type non linear inequalities can be found in [2]. In this paper we establish some new non-linear retarded integral inequalities in two independent variables which generalize the inequalities in [3, 4], this results can be used as handy tools to study the boundedness of solutions of differential-integral equations with the initial conditions. An application is given to illustrate the usefulness of our results.

**Keywords:** Retarded integral inequality, Boundness, Properties of solutions, Differential-integral equations.

**2010 Mathematics Subject Classification:** 45L05, 45G10.

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## Equilibrium for a static Bayesian game

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In this paper, we study a static Bayesian game in which at least some players are not completely informed of some other players' preferences, or some other characteristics of the other players that are relevant to their decision making.

Our contribution in this work concerns the study of a new equilibrium concept for this type of game, it is the Berge equilibrium. We have introduced a definition of Berge equilibrium for a static Bayesian game. Thus, we have established sufficient conditions to guarantee the existence of Bayesian Berge equilibrium. By using Schauder's fixed point theorem, we have transformed the problem of the existence of Bayesian Berge equilibrium in a problem of existence of a fixed point of an operator defined in the space of continuous functions in itself.

**Keywords:** Game theory, Bayesian game, Nash equilibrium, Berge equilibrium, Schauder fixed point theorem.

**2010 Mathematics Subject Classification:** 91A10, 26A51, 62C10.

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## New Integral Equations For The Hermite Polynomials

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The paper deals with some new integral equations for the normalized Hermite polynomials, where the intervening real functions does not depend on the index  $n$ .

**Keywords:** Linear functional, Hermite polynomials, Integral equation.

**2010 Mathematics Subject Classification:** 33C45, 42C05

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## Stability in higher-order nonlinear fractional differential equations

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**Abstract.** We give sufficient conditions to guarantee the asymptotic stability of the zero solution to a kind of higher-order nonlinear fractional differential equations. By using Krasnoselskii's fixed point theorem in a weighted Banach space, we establish new results on the asymptotic stability of the zero solution provided that  $f(t, 0) = 0$ . The results obtained here generalize the work of Ge and Kou [1].

**Keywords:** Higher-order fractional differential equations, fixed point theory, stability.

**2010 Mathematics Subject Classification:** 34K20, 34K30, 34K40.

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## Dynamic contact problem with total slip-rate-dependent friction for viscoelastic materials with long-term memory

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In this talk, we consider a mathematical model which describes the dynamic frictional contact between a deformable body and rigid foundation. We assume that the behavior of the body is described by a nonlinear viscoelastic constitutive law with long memory. The frictional contact is modeled with a simplified version of Coulomb's law in which the normal stress is prescribed and the coefficient of friction depends on the total slip-rate. We present the classical formulation of the problem, and derive a variational formulation which consists a second order evolutionary quasi-variational inequality for the displacement field. Then, we establish the existence and uniqueness result of weak solution. The proof is based on the Faedo-Galerkin method and Banach's fixed point theorem. Finally, we show a convergence result when the relaxation coefficients of long memory tend to zero.

**Keywords:** Banach's fixed point theorem, Faedo-Galerkin method, Total slip-rate, Variational inequalities, Viscoelastic with long-term memory.

**2010 Mathematics Subject Classification:** 47H10, 70M10, 58E35, 74D10.

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## Interest of the use of statistical methods in validation of food frequency questionnaire

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The main concern of any nutritionist is to develop a food frequency questionnaire, easy to use quickly and cheaply. This will allow identifying people at risk for nutritional diseases. The purpose of this study is to evaluate the validity of a previously developed semi-quantitative food frequency questionnaire (QFA), applicable to a large sample. For this 200 people aged between 18 and 50 years took part to this study. These participants responded in paper form to questions relating to their food consumption. The questionnaire was completed twice spaced by 5 weeks and comparisons were made using the Pearson test. The results obtained show for all food groups (fruits, vegetables, dairy products, sweet drinks, breads and sugar products) a good correlation between the two QFA (Pearson between 0.5 and 0.8). A low correlation is observed between the two QFA for weight and for the fat group (Pearson less than 0.5).

In conclusion the established QFA has a good correlation for most food groups. The limit of this questionnaire is observed in the concept of measurement to remedy more explicit photos can be used.

**Keywords:** Validation, Statistical methods, Food questionnaire.



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## Exact and Trajectory Controllability of First and Second Order Nonlinear Integro-Differential System: An Analytical and a Numerical Estimation

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The talk is divided in to two parts. In the first part, we have discussed an exact controllability of a neutral functional nonlinear system using a fixed point theorem. We have dropped the general assumption of the compactness of the semigroup and also defined the new phase space for the infinite delay. In the second part, we studied trajectory controllability of first and second order nonlinear integro-differential system using theory of monotone operators and tools of sine and cosine operators. We have then describe a general method for computing numerical approximations to control functions. We have illustrated both the methods with nonlinear integro-differential equations and a nonlinear hyperbolic equation in one and two spatial dimensions. Also, we have studied the fractional order system for both the parts.

**Keywords:** Exact Controllability, Phase space, Fixed point, Trajectory controllability, Second order Integro-differential system, Monotone operator theory, Cosine operators, Finite difference, Optimization

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## The mod $p$ homology of $F_4$ - gauge groups over $S^4$

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Let  $G$  be a compact, connected simple Lie group. The fact that  $\pi_3(G) = \pi_4(BG) = Z$  leads to the classification of principal  $G$  bundles  $P_k$  over  $S^4$  by the integer  $k$  in  $Z$ . The gauge group  $\mathcal{G}_k(G)$  acts freely on the space  $Map(P_k, EG)$  of all  $G$  equivariant maps from  $P_k$  to  $EG$  and its orbit space is given by the  $k$ -component of the space  $Map_k(S^4, BG)$  of maps from  $S^4$  to  $BG$ . Since  $Map(P_k, EG)$  is contractible, the classifying space of  $\mathcal{G}_k(G)$  is homotopy equivalent to  $Map_k(S^4, BG)$ . Then the number of homotopy types of  $\mathcal{G}_k(G)$  is finite [2].

In this paper we study the mod  $p$  homology of the gauge group associated with principal bundle of exceptional Lie group  $F_4$  by computing the Serre spectral sequence for the following fibration using the results in [1, 3]:

$$\mathcal{G}_k^b(F_4) \rightarrow \mathcal{G}_k(F_4) \rightarrow F_4$$

**Keywords:** exceptional Lie group  $F_4$ , gauge group, Eilenberg–Moore spectral sequence, Dyer–Lashof operation, Serre spectral sequence.

**2010 Mathematics Subject Classification:** 55R20, 55S12.

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## Two-dimensional inverse heat conduction problem in an unbounded domain

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We consider a two-dimensional inverse heat conduction problem in a region  $\{x > 0, y > 0\}$  with infinite boundaries which consists to reconstruct the boundary condition  $f(y, t) = u(0, y, t)$  on one side from the measured temperature  $g(y, t) = u(1, y, t)$  on accessible interior region. The numerical solution of the direct problem is realized by a boundary integral equation method [2]. The inverse problem is equivalent to an ill-posed integral equation [1]. For its approximation we use the regularization of Tikhonov after the mollification of the noised data. We show some numerical examples to illustrate the validity of the method.

**Keywords:** Inverse problem, Heat kernel, Integral equation.

**2010 Mathematics Subject Classification:** 35K05, 47A52.

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## Results on normal forms for Klein-Gordon lattices

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We study the Klein-Gordon (KG) lattice with periodic boundary conditions. It is an  $N$  degrees of freedom Hamiltonian system described by the Hamiltonian

$$H = \sum_{j \in \mathbb{Z}/N\mathbb{Z}} \left[ \frac{1}{2} p_j^2 + \frac{C}{2} (q_{j+1} - q_j)^2 + V(q_j) \right], \quad p_j = \dot{q}_j. \quad (1)$$

The constant  $C > 0$  measures the interaction to nearest neighbor particles (with unit masses) and  $V(x)$  is a non-linear potential of the form

$$V(x) = \frac{a}{2} x^2 + \frac{\beta}{2} x^4, \quad a > 0. \quad (2)$$

We assume that  $C$  is neither very large nor too small, hence we set  $C = 1$  which can be achieved by rescaling of  $t$ .

We show, assuming additionally  $a = 1$ , that in most cases the fourth order normal form of the periodic KG lattice is KAM nondegenerate Liouville integrable Hamiltonian. This implies the existence of many invariant tori in the original system (1), (2), on which the motion is quasi-periodic.

The integrability of the normal form of the KG lattice with Dirichlet boundary conditions is also discussed.

**Keywords:** Klein-Gordon lattice, Birkhoff normal form, KAM theory.

**2010 Mathematics Subject Classification:** 37J40, 70H06, 70H08.

## More results on quasi Cauchy sequences

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In this paper, we study on further results on quasi-Cauchy sequences and introduce new types of continuity. We obtain interesting results.

Keywords: Sequences, series, summability, continuity.

2010 Mathematics Subject Classification: 40A05, 40A35, 26A15.

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## On the spaces of almost convergent and strongly almost convergent double sequences

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The concepts of almost convergence and strongly almost convergence of double sequences were respectively defined by Móricz and Rhoades in [1] and Başarır in [2]. We denote by  $\mathcal{F}$ ,  $\mathcal{F}_0$ ,  $[\mathcal{F}]$  and  $[\mathcal{F}]_0$  the spaces of almost convergent, almost null, strongly almost convergent and strongly almost null double sequences, respectively. In this paper, we examine their general topological properties and give some important inclusion theorems. Also, we determine their alpha-, beta- and gamma-duals. Furthermore, we characterize some classes of four-dimensional matrix transformations related to those spaces.

**Keywords:** Double sequence, almost convergence, strongly almost convergence, dual space, matrix transformation.

**2010 Mathematics Subject Classification:** 46A45, 40C05.

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## On the difference spaces of almost convergent and strongly almost convergent double sequences

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The spaces  $\mathcal{F}$ ,  $\mathcal{F}_0$ ,  $[\mathcal{F}]$  and  $[\mathcal{F}]_0$  of almost convergent, almost null, strongly almost convergent and strongly almost null double sequences are studied by Çapan and Başar in [1]. In the present paper, we define the difference spaces  $\mathcal{F}(\Delta)$ ,  $\mathcal{F}_0(\Delta)$ ,  $[\mathcal{F}](\Delta)$  and  $[\mathcal{F}]_0(\Delta)$  of double sequences obtained as the domain of four-dimensional backward difference matrix  $\Delta$  in the spaces  $\mathcal{F}$ ,  $\mathcal{F}_0$ ,  $[\mathcal{F}]$  and  $[\mathcal{F}]_0$ , respectively. We examine general topological properties of those spaces and give some inclusion theorems. Furthermore, we determine their dual spaces.

**Keywords:** Double sequence, almost convergence, strongly almost convergence, difference sequence space, dual space.

**2010 Mathematics Subject Classification:** 46A45, 40C05.

## References

- [1] H. Çapan, F. Başar, On the spaces of almost convergent and strongly almost convergent double sequences, under communication.

## Necessary and Sufficient Conditions for the Second Order Differential Inclusions with Viable Constraint

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Optimization problems with viable constraints have wide area in application. However optimization problems for second-order differential inclusions with viable constraints have not been examined yet. In the present paper we derive the optimality conditions for the Mayer problem of second order discrete and differential inclusions with viable constraint. Applying necessary and sufficient conditions to problems with geometric constraints, optimality conditions for second order discrete inclusions are formulated. Using Locally Adjoint Mapping we conceive necessary and sufficient conditions for the optimality of the discrete approximation problem. Passing to the limit, sufficient conditions to the optimal problem are established.

**Keywords:** Differential inclusion, Mayer problem, Dual cone.

**2010 Mathematics Subject Classification:** 49K30, 49J52, 49M25, 90C31.

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## Graceful Labeling on Zigzag, Pari, and Vee Graph

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A simple, undirected, and finite graph  $G = (V, E)$  with  $p = |V|$  vertices and  $q = |E|$  edges is called graceful graph if the graph  $G$  can be labeled with an injective function  $f : V(G) \rightarrow \{0, 1, 2, \dots, q\}$  such that function  $f$  induces a bijection function  $f^* : E(G) \rightarrow \{1, 2, \dots, q\}$  defined as  $f(e = (u, v)) = |f(u) - f(v)|$ , for every edge  $e = (u, v)$  of  $G$ . This paper contains three results in graceful labeling. In this paper, zigzag graph  $Z_n$  and  $N_n$ , the join of complete bipartite graph  $K_{2,n}$ , pari graph and vee graph are proven as graceful graphs.

**Keywords:** Graceful labeling, Zigzag graph, Pari graph, Vee graph.

**2010 Mathematics Subject Classification:** 05C78

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## On Some Applications for Convex Set-Valued Mappings

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Let  $X, Y$  be normed linear spaces. If for each  $x \in X$  there is a corresponding set  $F(x) \subset Y$ , then  $F(\cdot)$  is called a set-valued map from  $X$  to  $Y$ . We denote this by  $F : X \rightrightarrows Y$ , for further information see [1-5]. In mathematics, we would like to know how a small change in a parameter of a given mathematical problem could affect the solution or solution set of the problem. So such a study is very important as many useful mathematical problems are usually approximately solved on the computer. Thus sensitivity analysis could guarantee the acceptability of the obtained approximate solution, based on certain allowed error on the parameters of the problem. For instance, the characterization of the variation of solution sets of optimization problems, partial differential equations, etc., is done thanks to set-valued maps. So, set-valued maps are essential tools in stability and sensitivity analysis of mathematical problems. Beside these, there are several other applications for set-valued maps. In this talk we will investigate the solvability conditions of some linear systems via the convex set-valued maps.

**Keywords:** Convex set-valued maps, sufficient conditions.  
**2010 Mathematics Subject Classification:** 58C06, 34A30.

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## On $g(\text{LC})$ - Spaces and $g(\text{KC})$ -space

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**Abstract:** In this paper we introduced a new definition and studied the properties of  $g(\text{LC})$ - spaces (which every  $g$  Lindelf subset is closed subset) and  $g(\text{KC})$ -spaces (which every  $g$ -compact subset is closed subset). Also, investigate their relationships with some other classes of topological spaces.

**Keywords** LC- spaces, KC- spaces,  $g$  closed

2010 Mathematics Subject Classification: 03E72, 46S40.

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## Necessary and Sufficient Conditions of Optimality for Convex Discrete and Differential Inclusions

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The problems considered in this talk are described by convex and non-convex third order discrete and differential inclusions. Our aim is to establish necessary and sufficient optimality conditions for Bolza problem of third order discrete, discrete-approximate and differential inclusions. Formulation of optimality conditions both for convex and non-convex discrete and differential inclusions plays a substantial role in incorporating the Euler-Lagrange and Hamiltonian type inclusions. The main concept of obtaining optimality conditions is based on the apparatus of locally adjoint mapping (*LAM*) and equivalence of *LAMs* theorems. Moreover, it appears that the use of the convex upper approximation (*CUAs*) for non-convex functions and local tents is very suitable for obtaining the optimality conditions for posed problems.

**Keywords:** Discrete-Differential inclusion, Euler-Lagrange, Transversality.  
**2010 Mathematics Subject Classification:** 49K20,49K24,49J52,49M25.

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## Free Time Optimization of Third Order Differential Inclusions with Endpoint Constraints

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This talk deals with the study of the optimal control theory with third order differential inclusions with a non-fixed time interval and endpoint constraints. Our main goal is to establish well-verifiable sufficient conditions of optimality for third order differential inclusions. Thus with the use of Euler-Lagrange and Hamiltonian type of inclusions and distinctive  $t_1$ -attainability conditions on the initial sets, the sufficient conditions are formulated. Applications of these results is illustrated by solving some linear control problem with third order differential inclusions.

**Keywords:** Free time, Attainability, Differential inclusion

**2010 Mathematics Subject Classification:** 49K21,34A60.

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## Further results on regularization and error estimates for a class of nonlinear abstract backward parabolic Cauchy problems

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The following research work is concerned with a class of nonlinear abstract backward Cauchy problems, known also as ill-posed problems. or final value problems (F.V.P.). This class of problems is known to be severely ill-posed. The instability problem is treated using a modified regularization method. The obtained family of regularised solutions is then shown to converge to the solution (in case of existence) of the original problem. Error estimates are also established.

Keywords: Ill-posed problem, quasi-reversibility value method, quasi-boundary value-methods.

2010 Mathematics Subject Classification: 35K05, 47J06, 47H10.

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## A note on lattice homomorphism

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The real algebra  $A$  is called a Riesz algebra or lattice-ordered algebra if  $A$  is a Riesz space such that  $ab \in A$  whenever  $a, b$  are positive elements in  $A$ . The Riesz algebra is called an  $f$ -algebra if  $A$  satisfies the condition that

$$a \wedge b = 0 \text{ implies } ac \wedge b = ca \wedge b = 0 \text{ for all } 0 \leq c \in A.$$

In an Archimedean  $f$ -algebra  $A$ , all nilpotent elements have index 2. Throughout this paper  $A$  will show an Archimedean semiprime  $f$ -algebra with point separating order dual. By definition, if zero is the unique nilpotent element of  $A$ , that is,  $a^2 = 0$  implies  $a = 0$ ,  $A$  is called semiprime  $f$ -algebra. It is well known that every  $f$ -algebra with unit element is semiprime. A linear operator  $T : E \rightarrow F$ , between two lattice ordered vector spaces  $E$  and  $F$  is said to be a lattice homomorphism if  $Tx^+ = (Tx)^+$  for all  $x \in E$ .

We prove that a positive operator,  $T : A \rightarrow B$ , between two semiprime  $f$ -algebras  $A$  and  $B$ , is a lattice homomorphism whenever there exists  $0 \leq a_0 \in A$  and  $0 \leq b_0 \in B$  such that  $T(a \wedge a_0) = T(a) \wedge b_0$ , for all  $a \in A$ .

Keywords: lattice homomorphism,  $f$ -algebra.

2010 Mathematics Subject Classification: 47B38.

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## Individual Claim Reserve Estimation using Generalized Linear Model

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In this paper, we will estimate claims reserve with Reserving by Detailed Conditioning (RDC) underlying Standard Generalized Linear Model(S-GLM). RDC method is an individual method of claims reserve estimation which involves detailed condition, so-called claim characteristics in calculation process. In the claim data of non-lifeinsurance, we get some information else and so-called background variable. The result estimate claim reserve by RDC method with background variableis not stable because many combination of calculation from each background variable, this is called segmentation. For estimate background variable as requirement RDC method, we can use linear model S-GLM. S-GLM is an method of estimation claim numbers by Poisson distribution and estimation claimseverities by Gamma distribution with coefficient of variation is constant.

**Keywords:** Individual Claim Reserve, Generalized Linear Model, Claim characteristics.

**2010 Mathematics Subject Classification:** 62P05

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## Application of mathematical models for analysis of severe plastic deformation processing of bulk materials

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Analytical approaches including upper bound and slip line methods have been widely used for analysis of deformation [1]. The growth of numerical methods for a similar purpose has been resulted in limited application of analytical methods with more and extended simplifications. Despite of this fact, these analytical approaches can help for more in-depth understanding of deformation which gives them the possibility to reserve their special place for analysis of deformation. In this research, application of mathematical modelling in analyzing plastic deformation of metals is discussed. Upper bound models are developed and used to investigate the deformation in equal channel angular pressing (ECAP), twist extrusion (TE), simple shear extrusion (SSE) and pure shear extrusion (PSE). The analyses are composed of hybrid combinations of computational and analytical approaches and consider effects of die design parameters, work hardening and friction. The models are experimentally verified using different dies and numerically using finite element method.

**Keywords:** Mathematical model, Analytical solution, Deformation.

**2010 Mathematics Subject Classification:** 81, 08.

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## The Construction of a Continuous Linear Representation From a Topological Group Into Topological Module Space Over Ring

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### Abstract

Let  $c$  be a continuous homomorphism from a topological group  $G$  into  $GLc(V)$  of all continuous bijective transformations. Homomorphism  $c$  like above is called a continuous linear representation from a topological group  $G$  into a topological vector space  $V$ . Because a module space is a vector space over a ring, not a field, we have construct a topological module space firstly. In this paper, we will construct a continuous linear representation from a topological group into a topological module space over ring.

**Keywords:** topological group, topological module space, continuous linear representation  
**2010 Mathematics Subject Classification:** 54CO5

## Robust HOS-based decision measure of moving objects and background

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The paper describes a scheme for moving objects in image sequence. The scheme adopts a distribution-based model approach to moving object detection. The methodology involves pattern classification using higher-order statistics (HOS) in a clustering framework. The proposed method effectively combines statistical knowledge about the class of moving objects with motion information. The unknown distribution of the image patterns of objects is approximately modeled using HOS information derived from sample images. Our proposed algorithm uses an HOS-based decision measure which is derived from a series expansion of the multivariate probability density function in terms of the multivariate Gaussian and the Hermite polynomial. A data clustering that uses HOS is proposed to facilitate the decision process. The higher-order closeness measure is used to classify test clusters as foreground or background.

**Keywords:** Motion estimation, Motion segmentation, Affine model, HOS-based decision measure, HOS-based k-means algorithm, Clustering.

**2010 Mathematics Subject Classification:** 68U10, 62H35, 94A08.

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## RBF solution of free convection in a semi-annulus involving EG-CuO nanofluid

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In this study, numerical simulation of hydrothermal behaviour of CuO nanoparticles based with ethylene glycol in a semi-annulus is investigated in presence of a magnetic source close to the heated inner cylinder. The non-dimensional governing equations are discretized by radial basis functions (RBFs) in space derivatives, and by Backward-Euler in time derivatives. The effects of dimensionless parameters are presented both in terms of streamlines, isotherms and vorticity contours, and average Nusselt number through the heated wall. The results show that augmentation in magnetic number accelerates the convective heat transfer at a small Rayleigh number, and at a larger heated inner cylinder. Convective heat transfer is inhibited with the rise of Hartmann number, and with the increase in Eckert number while it is enhanced with the increase in concentration of nanoparticles and Rayleigh number.

**Keywords:** Radial Basis Functions, Ethylene Glycol, Magnetic source.

**2010 Mathematics Subject Classification:** 65M70, 76M22, 82D80.

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## A variation on strongly lacunary delta ward continuity in 2-normed spaces

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A sequence  $(x_k)$  of points in a subset  $E$  of a 2-normed space  $X$  is called strongly lacunary quasi-Cauchy, or  $N_\theta$ -quasi-Cauchy if  $(\Delta x_k)$  is  $N_\theta$ -convergent to 0, that is  $\lim_{r \rightarrow \infty} \frac{1}{h_r} \sum_{k \in I_r} \|\Delta x_k, z\| = 0$  for every fixed  $z \in X$  and it is denoted by  $N_\theta - \lim_{k \rightarrow \infty} \|\Delta x_k, z\| = 0$ . A function defined on a subset  $E$  of  $X$  is called strongly lacunary ward continuous if it preserves  $N_\theta$ -quasi-Cauchy sequences, i.e.  $(f(x_k))$  is an  $N_\theta$ -quasi-Cauchy sequence whenever  $(x_k)$  is. The purpose of this study is to introduce the concept of strongly lacunary delta ward continuity in 2-normed spaces and prove some related theorems.

**Keywords:** Strongly lacunary ward continuity, quasi-Cauchy sequences, continuity, 2-normed space.  
**2010 Mathematics Subject Classification:** 40A05, 40G15; 26A05, 26A15.

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## On Ideal Acceleration Convergence of Double Sequences

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**Abstract:** The main aim of this article is to introduce the concept of ideal acceleration convergence of double sequences in Pringsheim's sense. We prove the decomposition theorem for ideal acceleration convergence of double sequences and some theorems related to that concept have been established using the four dimensional matrix transformations.

We provided some examples, where the results of acceleration convergence fails to hold for the ideal cases.

**Keywords:** Converging faster, acceleration field, I-convergence, double natural density.  
**2010 Mathematics Subject Classification:** 40C05, 42B15.

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## $\Delta$ –Convergence on Partial Metric Space

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**Abstract:** The concept of partial metric space is a minimal generalization of a metric space  $(X; d)$ , where for each  $x \in X$ ;  $d(x; x)$  does not need to be zero, in other terms is known as non-self distance. In this paper we defined some new definitions on partial metric space using by first difference of a sequence  $(x_k)$  in partial metric space and examined some properties of these definitions.

**Keywords:** Metric space, partial metric space, difference sequence.

**2010 Mathematics Subject Classification:** 40A05, 40C05

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## On lacunary $d$ -statistical convergence of order $\alpha$

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The statistical convergence of real or complex valued sequences was first introduced by Fast [2], but the idea of statistical convergence goes back to Zygmund [4] in the first edition of his monograph published in Warsaw in 1935. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. In this study using a lacunary sequence, we introduce the concepts of lacunary  $d$ -statistical convergence sequences and lacunary strong  $dp$ -Cesàro summable sequences in general metric spaces.

**Keywords:** Statistical convergence, Cesàro summability.

**2010 Mathematics Subject Classification:** 40A05, 40C05, 46A45.

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## An iterative algorithm for split generalized equilibrium problem for new mappings

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In this paper, we introduce and study a general iterative algorithm to approximate a common solution of split generalized equilibrium problem, variational inequality problem and fixed point problem for a finite family of nonexpansive mappings in real Hilbert spaces. Further, we prove a strong convergence theorem for the sequences generated by the proposed iterative scheme. Finally, we derive some consequences from our main result. The results presented in this paper extended and unify many of the previously known results in this area.

**Keywords:** Split generalized equilibrium problem, Variational inequality problem, Fixed-point problem, Nonexpansive mapping.

**2010 Mathematics Subject Classification:** Primary 46C99, 49J30; Secondary 47H10, 47H17, 90C99.

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## Precipitation phenomena and kinetics in Mg-Al Alloys

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This study investigated the effect of aging on the precipitation and kinetics of second phase  $Mg_{17}Al_{12}$  in Mg-Al alloy, using X-ray diffraction, microhardness measurements and differential scanning calorimetry analysis (DSC). With the last instrument, the all samples were heated from room temperature to 400 °C, at heating rates of 10 to 30 °C/min. The results were supplemented by measured the average of activation energies, using isothermal treatments by JohnsonMehlAvrami (JMA) methods and by non-isothermal treatments using Ozawa, Boswell, Kissinger and Mahadavan methods, were around 67 and 60 kJ/mol. The numerical factor  $m$  and the Avrami parameter  $n$  is estimate to be approximately equal to 3 and 2.79 respectively. This value corresponding that the bulk nucleation with constant number of nuclei was dominant in three-dimensional (polyhedron) controlled by interface reaction.

**Keywords:** DSC;  $Mg_{17}Al_{12}$ ; precipitation; activation energy

**2010 Mathematics Subject Classification:** 74A50, 80Axx, 82B26.

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## Tauberian theorems for the product of weighted and Cesàro summability methods for double sequences

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We obtain necessary and sufficient conditions, under which convergence of a double sequence in Pringsheim's sense follows from its weighted-Cesàro summability. These Tauberian conditions are one-sided or two-sided if it is a sequence of real or complex numbers, respectively.

**Keywords:** Tauberian theorems, double sequences, weighted-Cesàro summability.  
**2010 Mathematics Subject Classification:** 40E05, 40G05.

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## Some Tauberian theorems for weighted means of double integrals

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Let  $p(x)$  and  $q(x)$  be nondecreasing continuous functions on  $[0, \infty)$  such that  $p(0) = q(0) = 0$  and  $p(x), q(x) \rightarrow \infty$  as  $x \rightarrow \infty$ . For a continuous function  $f(t, s)$  on  $\mathbb{R}_+^2 = [0, \infty) \times [0, \infty)$  we define its double integral by  $F(x, y) = \int_0^x \int_0^y f(t, s) dt ds$  and its weighted mean of type  $(\alpha, \beta)$  by

$$t_{\alpha, \beta}(x, y) = \int_0^x \int_0^y \left(1 - \frac{p(t)}{p(x)}\right)^\alpha \left(1 - \frac{q(s)}{q(y)}\right)^\beta f(t, s) dt ds$$

where  $\alpha > -1$ , and  $\beta > -1$ . We say that  $\int_0^\infty \int_0^\infty f(t, s) dt ds$  is integrable to  $L$  by the weighted mean method of type  $(\alpha, \beta)$  determined by the functions  $p(x)$  and  $q(x)$  if  $\lim_{x, y \rightarrow \infty} t_{\alpha, \beta}(x, y) = L$  exists. We prove that if  $\lim_{x, y \rightarrow \infty} t_{\alpha, \beta}(x, y) = L$  exists for some  $\alpha > -1$  and  $\beta > -1$ , then  $\lim_{x, y \rightarrow \infty} t_{\alpha+h, \beta+k}(x, y) = L$  exists for all  $h, k > 0$ . Finally, we prove that if  $\int_0^\infty \int_0^\infty f(t, s) dt ds$  is integrable to  $L$  by the weighted mean method of type  $(1, 1)$  determined by the functions  $p(x)$  and  $q(x)$  and conditions  $\frac{p(x)}{p'(x)} \int_0^y f(x, s) ds = O(1)$  and  $\frac{q(y)}{q'(y)} \int_0^x f(t, y) dt = O(1)$  hold, then  $\lim_{x, y \rightarrow \infty} F(x, y) = L$  exists.

Keywords: Divergent integrals, Weighted mean integrability, Tauberian theorems and conditions  
 2010 Mathematics Subject Classification: 40A10, 40C10, 40D05.

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## More Maximal Arcs and Their Codes

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In a recent work [3], eight maximal arcs of degree 4 in some of the projective planes of order 16 were found. Using a similar technique, we found 8 more maximal  $(52,4)$ -arcs. We studied the properties of linear codes associated with all known maximal arcs of degree 4 (including newly discovered ones), and we see that these codes provide new connections between planes of order 16. Details of this study will be provided.

**Keywords:** Maximal Arcs, Projective Planes, Linear Codes.

**Mathematics Subject Classification:** 94B99.

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## **Statistical optimisation of lactic bacteria growth medium by Plackett and Burman design and central composit design**

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Statistical tools, Plackett and Burman design (PBD) and central composit design (CCD) were used to select and optimise medium parameters for lactic bacteria gorwth. These designs solve the problem of conventional process studies wich are not only time consuming and expensive but also failed to consider the interactions between different factors.

**Keywords:** Statistical Optimisation, Plackett and Burman Design, Central Composit Design.  
**2010 Mathematics Subject Classification:** 92B15, 62P99

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## An $st$ -Connected Vertex Separator problem

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In this paper, we give some basic notions concerning the  $st$ -connected vertex (edges) separator problem( $st$ -CVS(CES) problem), then we give three integer programming formulations for this problem. Also, we introduce a heuristic to solve this problem, along with an extensive computational study is presented.

**Keywords:**graph partitioning, connected separator, heuristic.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## Study of mechanical and thermal properties behavior of the PS/TiO<sub>2</sub> composite

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The study of mechanical and thermal properties behavior of the composite PS/TiO<sub>2</sub> tempered aims to establish relationships between these properties and the quenching temperature. The mechanical properties of the composite polystyrene-titania (PS/TiO<sub>2</sub>) can be improved by introducing residual stresses. Tempering generates compressive surface stress, which leads to strengthening. The composite PS/TiO<sub>2</sub> is recognized as a hard and a brittle thermoplastic polymer. It was observed that the hardening carried out at defined temperatures, residual stress relaxation mode of the polymer produced a transition to ductile behavior of residual stresses resulting in the increase in impact strength, elongation at break and lower elastic modulus, and density. The quenching method is a simple and effective way to generate thermal stresses in the composite PS/TiO<sub>2</sub>. This process leads to structural changes and mechanical reinforcement. In this case, quenching is only effective for thicknesses above 1 mm. The presence of compressive residual stresses prevents the diffusion of methanol.

**Keywords:** quenching temperature, PS/TiO<sub>2</sub>, brittle, residual stress.

**2010 Mathematics Subject Classification:** 74F05 , 74A40, 82D60.



## Determination of Magnesium Arsenide Melting Point by MD Simulation Method

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In this study, melting point of magnesium arsenide structure was determined with the two phase simulation box MD simulation method. Calculation of melting temperature in single-phase simulation boxes is often results inaccurate outputs. As a result of such calculations melting temperature is overestimated. For this reason, two-phase simulation method was used to avoid such a problem [1]. Two different simulation boxes were equilibrated separately in solid and liquid state phases, 27°C and 1627°C respectively. These two equilibrated simulation boxes are put together side by side. NPH ensemble was applied to two phase supercell. At the end of a series of simulations, with different initial temperatures, melting temperature was calculated about  $\approx 1297^\circ\text{C}$  with the available interaction potential parameters. Experimental melting temperature is above  $1200^\circ\text{C}$  [2].

Keywords: Melting Point, Phase Transformations, MD Simulation.

2010 Mathematics Subject Classification: 74A50, 82D15, 37N30.

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## The Static Structure of Molten Magnesium Antimonide

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The static structure of molten magnesium antimonide was calculated by using hypernetted-chain theory of liquids. The Ornstein-Zernike equation had been solved numerically. A semi-empirical interaction potential was used for the numerical calculation. In our previous studies, interaction potential was developed for magnesium antimonide with MD simulation techniques and verified by comparing with experimental results [1]. Bhatia-Thornton partial structure factors were calculated at 1277°C. At this temperature  $Mg_3Sb_2$  was completely in a molten state because experimental melting temperature of magnesium antimonide is about 1245°C [2]. Moreover melting point was calculated about 1027°C by MD simulation with the present interaction potentials [1]. Partial structure factors like, density-density, concentration-concentration and density-concentration functions, ( $S_{NN}$ ,  $S_{CC}$ ,  $S_{NC}$ ) were calculated. Pre-peak in the low-k region was observed in  $S_{NN}$  which displayed ionic conductivity of components were very different from each other.

Keywords: Static Structure, Hypernetted-chain theory of liquids, Numerical Methods.  
2010 Mathematics Subject Classification: 82-08, 82D15, 82B80.

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## Calculation of Mechanical Properties of Low Temperature Form $Mg_3As_2$

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Magnesium arsenide,  $Mg_3As_2$ , is a material that has not been studied much in the literature. One of the reasons for this is that although it is a semiconductor material, there may be less interest due its toxic effect. The lack of interest of the experimental groups has probably influenced the theoretical groups. In this study mechanical properties of low temperature form of magnesium arsenide was investigated by MD simulation and numerical methods [1]. These static properties were calculated by BFGS energy minimization method. Mechanical properties like, bulk–shear moduli and elastic constants like,  $C_{11}$ ,  $C_{12}$  and  $C_{44}$  were calculated and results were compared with the available first–principles studies in the literature which were in good agreement.

**Keywords:** Elastic Constants, Bulk modulus, Numerical Methods.

**2010 Mathematics Subject Classification:** 82-08, 82B80.

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## Growth of solutions of linear differential equations near a singular point

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In this talk, we investigate the growth of solutions to linear differential equations whose coefficients are meromorphic or analytic in the closed complex plane except at a finite singular point. For that we will use the value distribution theory of meromorphic functions due to Rolf Nevanlinna with adapted definitions. We will see the similarities and differences between this investigation and the classical theory concerning the growth of solutions in the complex plane and in the unit disc.

**Keywords:** Linear differential equations, local growth of solutions, singular point.

**2010 Mathematics Subject Classification:** 34M10, 30D35.

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## **Assessment the Relationship between High Level of Cholesterol and Heart Disease in Baquba Teaching Hospital by using the Method of Fuzzy Regression**

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We often use regression analysis to assess and predict the relationship between the variables, specifically the relationship between dependent variable and one or more of independent variables. Today, new techniques are used to evaluate the relationship between variables that show the effect of more than one independent variable in different proportions on a dependent variable, meaning that more than one independent variable is involved in determining the effect on the dependent variable. These techniques have been widely used in applied sciences, social sciences, medicine, economy, industry and other fields. The research aims to assess the relationship between high level of cholesterol and heart diseases and diagnose the most important factors affecting the heart disease of people with high cholesterol in Baquba Teaching Hospital by using the method of fuzzy regression depending on variables of a real data. Finally, Mean Squared Error, F test, t test and determination of coefficient are provided to compare and illustrate the results that summarized in tables.

Keywords Fuzzy Regression ,Fuzzy Least Square method , Least Square method  
2010 Mathematics Subject Classification: 03E72, 46S40.

## Almost meshed locally connected continua are $(n, m)$ -fold hyperspace suspension determined

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For a metric continuum  $X$  and  $n \in \mathbb{N}$ , we consider the hyperspaces  $C_n(X)$  (respectively,  $F_n(X)$ ) of all nonempty closed subsets of  $X$  with at most  $n$  components (respectively,  $n$  points). Given positive integers  $n$  and  $m$  such that  $m \leq n$ , let  $HS_m^n(X)$  be the quotient space  $C_n(X)/F_m(X)$  with the quotient topology. In this talk, for each  $n \in \mathbb{N} - \{1, 2\}$ ,  $m \in \mathbb{N} - \{1\}$  and  $m < n$ , we prove that: if  $X$  and  $Y$  are almost meshed locally connected continua and  $HS_m^n(X)$  is homeomorphic to  $HS_m^n(Y)$ , then  $X$  is homeomorphic to  $Y$ .

**Keywords:** Continuum, hyperspace suspension, unique hyperspace.

**2010 Mathematics Subject Classification:** Primary 54B20, Secondary 54F15.

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## On the class of disjoint limited completely continuous operators

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We introduce and study new class of sets (almost L-limited sets). Also, we introduce new concept of property in Banach lattice (almost Gelfand-Phillips property) and we characterize this property using almost L-limited sets. On the other hand, we introduce the class of disjoint limited completely continuous operators witch is a larger class than that of limited completely continuous operators, we characterize this class of operators and we study some of its properties.

**Keywords:** limited set, almost L-limited set, Gelfand-Phillips property, almost Gelfand-Phillips property.

**2010 Mathematics Subject Classification:** 46B42, 47B60, 47B65.

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## A New Technique of The $q$ -Homotopy Analysis Method for Solving Non-Linear Initial Value Problems

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In this paper, a new procedure of the  $q$ -homotopy analysis technique (NT $q$ -HAM) was submitted for solving non-linear initial value problems. The NT $q$ -HAM contains just a single convergence control parameter  $\alpha$ . To show the dependability and proficiency of the technique, this approach is applied to solve two non-linear IVPs, and the outcomes uncover that the NT $q$ -HAM is more general of the homotopy perturbation method (HPM) [22] and the HPM is only special case of the NT $q$ -HAM when

**Keywords:** Fractional order, Ordinary differential equations, Haar wavelet  
**2010 Mathematics Subject Classification:** 03E72, 46S40.



## Formulation of Approximate Mathematical Model for Incoming Water to Some Dams on Tigris and Al-Fourat Rivers Using Spline Functions

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In this paper, we formulate three mathematical model using spline functions such as linear, quadratic and cubic to approximate the mathematical model for incoming water to some dams. We will implement this model on dams of both rivers, dams on the Tigris river are Mosul and Amara while dams on the Euphrates river are Hadetha and Al-Hidya.

Keywords: Spline Function, Approximate mathematical model, Incoming water, Dams.  
2010 mathematics subject classification: 41A15,34K28,65C20

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## A note on generalized solutions of differential inclusions

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We consider the following differential inclusion in general form:

$$\dot{x}(t) \in F(t, x(t)) \quad (1)$$

where  $x(\cdot)$  is an absolutely continuous ( $AC$ ) function from  $[0, T]$  to  $\mathbb{R}^n$ , while  $F : [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a set-valued function. We say that the function  $x(\cdot) \in AC([0, T], \mathbb{R}^n)$  with  $x(0) = x_0$  satisfying (1) almost everywhere is a solution of inclusion (1) with the initial condition  $x_0$ .

In this paper, we study the inclusion (1) where  $F$  is defined in a special form by using the notions of weak differentiability for lipschitz function. Existence of solution of the inclusion and some properties of solution are obtained.

**Keywords:** weak differentiability, differential inclusion.

**2010 Mathematics Subject Classification:** 28B20, 49J52.

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## Variations on quasi Cauchy sequences in metric spaces

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For a fixed positive integer  $p$ , a sequence  $(x_n)$  in a metric space  $X$  is called  $p$ -quasi-Cauchy if  $(\Delta_p x_n)$  is a null sequence where  $\Delta_p x_n = d(x_{n+p}, x_n)$  for each positive integer  $n$ . A subset  $E$  of  $X$  is called  $p$ -ward compact if any sequence  $\mathbf{x} = (x_n)$  of points in  $E$  has a  $p$ -quasi-Cauchy subsequence. A subset of  $X$  is totally bounded if and only if it is  $p$ -ward compact. A function  $f$  from a subset of  $X$  into  $X$  is called  $p$ -ward continuous if it preserves  $p$ -quasi Cauchy sequences, i.e.  $(f(x_n))$  is a  $p$ -quasi Cauchy sequence whenever  $(x_n)$  is a  $p$ -quasi Cauchy sequence of points of  $E$ . A function  $f$  from a totally bounded subset of  $X$  into  $X$  preserves  $p$ -quasi Cauchy sequences if and only if it is uniformly continuous, i.e.  $(f(x_n))$  is a  $p$ -quasi Cauchy sequence whenever  $(x_n)$  is a  $p$ -quasi Cauchy sequence of points of  $E$ .

**Keywords:** quasi cauchy sequence, statistical convergence, compactness.

2010 Mathematics Subject Classification: 40A05, 26A15, 42A65, 54C30.

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## Investigation of the heat treatment effect on opto-electronic properties of CdS /Si Junctions

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Cadmium Sulfate is considered as the second lowest cost material after Si in the field of thin-film solar cells and photovoltaic application, this work focused on the development of optoelectronic properties of the CdS/Si Junctions. Thin film of Cds/Si junction has been prepared by use a spray pyrolysis technique, followed by annealing at various temperatures (400-600 oC), in order to find out the effect of annealing condition on the properties and identify the best heat treatment to optimize the films features. The results have confirmed the structure was cubic and hexagonal wurtzite with preferential orientation in the H(002) and C(111) direction around the c-axis. Increasing the annealing temperature led to the emergence of new peaks in X-Ray pattern which means there was change in structure has occurred. The electrical and optical properties were studied by recording the value of the junction capacitance as a function of reverse bias voltage at 40 KHz frequency and I-V measurements respectively, these results show the high temperature treatment increases the built in potential from 0.15 V to 0.62V, and also expand the width of the depletion layer from 66.6nm to 143.81nm.

Keywords built in potential, spray pyrolysis, depletion layer

2010 Mathematics Subject Classification: 03E72, 46S40.

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## Coupled and common coupled fixed point theorems in soft metric and fuzzy soft metric spaces

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In this paper, we prove coupled fixed point theorems for some mappings by using the concepts  $q$ -contractive soft mapping and  $R$ -weakly commuting soft mappings in soft metric spaces and fuzzy soft metric spaces.

**Keywords** soft metric space, fuzzy soft metric space, coupled fixed point theorems.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## New Weyl-Type Theorems for Direct Sums

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**Abstract.** In this paper, we continue to study the new Weyl-type Theorems and the stability of it, for the orthogonal direct sum  $\oplus_{i=1}^n T_i$ , where  $T_1, T_2, \dots, T_n$  are bounded linear operators acting on Banach spaces  $X_1, X_2, \dots, X_n$  respectively such that  $n \geq 2$ . Among other results, the property  $(UWE)$  in general is not preserved under direct sum, in this note we explore certain sufficient conditions on  $T_1, T_2, \dots, T_n$ , so that the property  $(UWE)$  holds for the direct sum  $\oplus_{i=1}^n T_i$ .

**Keywords:** Property  $(UW\Pi)$ , property  $(UWE)$ , direct sums, B-Weyl spectrum.

**2010 Mathematics Subject Classification:** Primary 47A53, 47A10, 47A11.

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## Harmonic analysis associated to the multivariate Laguerre function

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The aim of this talk is to associate an harmonic analysis for multivariate Laguerre's functions  $\mathcal{L}_m^\alpha(x)$ ,  $x = (x_1, \dots, x_n) \in [0, +\infty[^n$ . We consider the kernel, defined on  $\mathbb{K} = [0, +\infty[^n \times \mathbb{R}$ , by :

$$\Psi_{m,\lambda}^\alpha(x, t) = e^{i\lambda t} \mathcal{L}_m^\alpha(|\lambda| x^2), \quad (m, \lambda) \in \mathbb{N}^n \times \mathbb{R}, \quad \text{where } x^2 = (x_1^2, \dots, x_n^2).$$

We begin by establishing a product formula for these kernels. Then we introduce a generalised Fourier-Laguerre transform as follows :

$$\mathfrak{F}(f)(m, \lambda) = \frac{1}{\pi^n} \int_{\mathbb{K}} f(x, t) \Psi_{m,-\lambda}(x, t) \prod_{k=1}^n \frac{x_k^{2\alpha_k+1}}{\Gamma(\alpha_k+1)} dx dt, \quad (m, \lambda) \in \mathbb{N}^n \times \mathbb{R}.$$

We provide the following theorems.

**Theorem 1.** (Plancherel theorem)

For all  $f \in L_\alpha^1(\mathbb{K}) \cap L_2^\alpha(\mathbb{K})$ , we have the following Plancherel formula :

$$\|f\|_{L_\alpha^2(\mathbb{K})}^2 = \|\mathfrak{F}(f)\|_{L_\alpha^2(\mathbb{N}^n \times \mathbb{R})}^2.$$

**Theorem 2.** (Paley-Wiener theorem)

The generalized Fourier-Laguerre transform  $\mathfrak{F}$  is a topological isomorphism from  $\mathcal{D}_*(\mathbb{K})$  onto  $\mathcal{S}_{\mathbb{H}}(\mathbb{N}^n \times \mathbb{R})$ .

For Laguerre's function on  $\mathbb{R}_+$ , a harmonic analysis can be find in [1].

**Keywords:** Fourier-Laguerre transform, Plancherel theorem, Paley Wiener theorem.

**2010 Mathematics Subject Classification:** 41A27, 42B10, 44A05.

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## A study on variations on strongly lacunary quasi Cauchy sequences

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In this paper, we study on further results on strongly lacunary quasi-Cauchy sequences and strongly lacunary ward continuity. We obtain interesting results.

**Keywords:** Sequences, series, summability, continuity.

**2010 Mathematics Subject Classification:** 40A05, 40A35, 26A15.

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## Weibull Stress Strength Reliability Bayesian Estimation

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In this paper , we consider the estimation reliability when  $x$  and  $y$  are two independent Weibull random variables with parameters (  $\alpha$ ,  $\beta$  ) and (  $\alpha$ ,  $\lambda$  ) respectively. A stress strength model defines life of a component with strength  $x$  and stress  $y$  . Bayesian analysis has been considered using [ Weighted, Quadratic and Entropy ] loss functions under two informative and non-informative priors [Gamma and Extension of Jeffery ] with singly type II censored , then obtained empirical Bayes estimators of reliability corresponding to Gamma prior distribution. A simulation study worked to generate random variables for reliability by different experiments based on different sample sizes and different values of parameters (  $a, b$  ) with replicate  $q = 1000$  , then comparisons have been made between the reliability estimators using MSE criteria .

**Keywords:** Weibull Stress, Bayesian Estimation, MSE criteria

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## Hypothesis Test of Geographically Weighted Regression Models Using Bootstrap

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### Abstract

Some statistical tests to find out whether some of the resulting coefficients have an influence on Geographically Weighted Regression (GWR) models play an important role in exploring spatial non-stationary relationships in regressions. Given some of the deficiencies of existing inferential methods, a residual-based bootstrap test is proposed to test the constant coefficient hypothesis in the GWR model by calculating its p-value. The proposed test is free of assumptions such as normality. Furthermore, the theoretical result of this bootstrap tests is applied to some real economics data sets of Indonesia that is fitted Geographically Weighted Regression (GWR) models.

**Keywords:** GWR, regression coefficient inference, bootstrap, p-value

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## Linear Stability Analysis of An Integro-difference System

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In this talk, we investigate the dispersal-driven instabilities that arise in a system of integro-difference equations. These equations have been built around redistribution kernels and dispersal has been modeled using convolution integrals. It is shown that the spatially homogeneous equilibrium of this system becomes unstable for some kernel functions and parameter values by performing a linear stability analysis.

**Keywords:** Integro-difference system, pattern formation, linear stability analysis.

**2010 Mathematics Subject Classification:** 35B36, 37C75.

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## Some set-open and uniform topologies on $C(X)$

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Let  $C(X)$  be the set of all real-valued continuous functions on a topological space  $X$  and  $\alpha$  a family of real compact subsets of  $X$ . We study some comparison between uniform and set-open topologies on  $C(X)$  defined by the family  $\alpha$ .

**Keywords:** Uniform topology, Set-open-topology, Real compactness

**2010 Mathematics Subject Classification:** 54C35

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## Intelligent Control of Manipulator Robot

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This work deals with application of learning machines approaches for estimation of dynamic model of manipulator robot, in order to use it in control laws. Indeed, most of control laws need the exact robot model to give good performances. The complexity of the robot manipulator and its strong non-linearity makes its parameters hard to be known. Many studies have been developed in the literature for such problematic, and many control tools are proposed.

In this paper, we focus on the use of support vector machine technique to estimate the model of nonlinear system to control the system during its evolution in its workspace.

**Keywords:** Nonlinear system, Manipulator robot, Support vector machine.

**2010 Mathematics Subject Classification:** 68T40, 70Q05, 93C85.

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## Application of Sequential Probability in Reliability Test (SPRT)

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In order to make a final decision on the rejection or acceptance of a hypothesis, requires taking a number of observations until reaching a decision to reject , accept or examine each sample sequentially. A minimum number of observations may be required to reach the final decision regarding the rejection or acceptance of the present batch for the inspection. However, the inspector or researcher must include in his mind that the costs of the inspection depend on the number of units inspected , ie, taking the samples at once may be costly and take time and effort. In this paper, we discuss the use of sequential probability ratio test (SPRT),to measure the reliability samples were taken in a sequential manner and their results were evaluated at each stage. and reaching an early decision to reject or accept the batch. Thus reducing the time and effort. The SPRT was applied to measure the reliability of the electric mixer. It requires high efficiency with few error from the SPRT and this test is necessary to requires more information to in hence decision to accept or reject the batch at test sample size .

**Keywords:**Sampling inspection, Sequential inspection, Sequential probability ratio test (SPRT), Reliability .  
**2010 Mathematics Subject Classification:** Applied Mathematics,Statistics, Mathematical Statistics.

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## Chain Code Algorithm Enhancement

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Abstract Chain code techniques are wide used in shape recognition, and it considered as standard input form for many algorithms of shape analysis and recognition. All concern enhancement and development the work of chain code studies were limited to treat and solve the coding problems such as start point, change of size and rotation, so there is need to develop the chain code algorithm in other directions such as making it works on open shapes beside closed shapes and solving the problem of length of coding digits. This paper presents study the chain code algorithm and fixing some of its weakness points, where the problem of open shapes solved besides solving the problem of length of code digits by using RLE method to compress the data. The enhanced algorithm is applied on many images and all shapes inside used images recognized. Matlab is used for this work.

Keywords Chain Code, RLE method, Matlab

2010 Mathematics Subject Classification: 03E72, 46S40.

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## Approximation by a generalization of the Jakimovski-Leviatan operators

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In this paper, we introduce a Kantorovich type generalization of Jakimovski-Leviatan operators constructed by A. Jakimovski and D. Leviatan (1969) and the theorems on convergence and the degree of convergence are established. Furthermore, we study the convergence of these operators in a weighted space of functions on a positive semi-axis.

**Keywords:** Jakimovski-Leviatan operator, Lipschitz class, weighted modulus of continuity, weighted spaces.

**2010 Mathematics Subject Classification:** 41A25.

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## On some geometric structures of 4–dimensional manifolds

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This study is devoted to investigate some geometric structures in smooth manifolds of dimension 4 admitting a metric of any signature. First of all, some fundamental concepts of neutral, Lorentz and positive definite signatures are given on these manifolds. Then, a brief information on the classification of Riemann curvature tensor and 2-forms is provided by considering such metric signatures. Some properties of 2-forms are obtained when the manifold admits a neutral metric signature. After that Weyl conformal and Weyl projective tensors are considered for the neutral signature case and some results are found for these special tensor fields. These results are then associated with the curvature types and the holonomy theory. Finally, similar problems are examined for the other metric signatures (that is, Lorentz and positive definite signatures) and some theorems are proved.

**Keywords:** 4-dimensional manifold, metric signature, tensor field.

**2010 Mathematics Subject Classification:** 53C29, 53C50, 53B30.

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## The stability of a generalized trigonometric functional equation

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In this talk, we will study the reduced stability of the cosine and the sine functional equation from the generalized trigonometric functional equation

$$f(x+y) - g(x-y) = \lambda \cdot h(x)k(y) \quad \lambda : \text{constant},$$

which  $f, g, h, k : G \rightarrow A$  can be considered the mixed functional equation of the sine and cosine functions, which is represented by the mixed exponential type functional equations, hyperbolic cosine (sine) functions.  $G$  is Abelian group and  $A$  is a unital normed algebra.

**Theorem 1.** Assume that  $\varphi(x, y)$  is bounded as a function of  $x$  for each  $y \in G$ , and that  $f, g, h, k : G \times G \rightarrow A$  satisfy the inequality

$$\|f(x+y) - g(x-y) - \lambda h(x)k(y)\| \leq \varphi(x, y) \quad (1)$$

for all  $x, y \in G$  with  $\lambda > 0$  is a constant. If there exists a sequence  $\{x_n\}$  in  $G$  such that

$$\lim_{n \rightarrow \infty} \|h(x_n)^{-1}\| = 0,$$

then  $k$  satisfies

$$k(x+y) + k(x-y) = \lambda k(x)l_h(y) \quad (2)$$

for all  $x, y \in G$ , in which,  $l_h : G \rightarrow F$  is an even function such that  $l_h(0) = 2\lambda^{-1}$ .

**Keywords:** stability, superstability, , trigonometric (cosine, sine) functional equation

**2010 Mathematics Subject Classification:** 39B82, 39B52.

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## Stability and Hopf bifurcation of 3D predator-prey models with Allee effect via computational algebra

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In this work, we give a compared analysis for two prey-one predator models and one prey-two predator models with Allee effect in the growth functions and in nonlinear functional responses. First, we analyze the stability properties of the systems under consideration to understand the impact of the Allee effect in the behavior of the different generalized versions of the models. Then, we give the parameter conditions for Hopf bifurcation to occur for the systems. We investigate the algebraic properties of the systems by finding all algebraic invariants of degree two and three. We reduce the systems to two dimensional systems by using an appropriate algebraic invariant in order to find the parameter conditions for the existence of a stable limit cycle. We find formulae for the first Lyapunov coefficients to investigate the stability of Hopf bifurcations. Finally, we present our algebraic findings by giving some numerical examples.

**Keywords:** Stability, Hopf bifurcation, algebraic invariant.

**2010 Mathematics Subject Classification:** 37G10, 65P30, 13A50.

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## Uniqueness of hyperspaces

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Given a topological space  $Z$  there are several ways to construct a new space  $R(Z)$  from  $Z$ . Examples of this situation are: products, spaces of maps to the real line, group of homeomorphisms, hyperspaces, etc. A natural question is if we can recover the space  $Z$  when we know the space  $R(Z)$ . In order to reach this aim, the space  $R(Z)$  must be unique. That is, it must come only from one  $Z$ . In this talk we survey what has been done on this direction, for a continuum  $Y$ , when the structure  $R(Y)$  is one of the hyperspaces  $2^X$ ,  $C_n(X)$  or  $F_n(X)$ , when  $n$  is a positive integer.

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## A Note On The Structure Constants Of Leibniz Algebras

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Leibniz algebras are certain generalization of Lie algebras. In this note, our main goal is to focus on structure constants of Leibniz algebras and to give some properties of the structure constants of Leibniz algebras. In particular, some conditions are investigated for non-Lie Leibniz algebras.

Keywords: Lie algebra, Leibniz algebra, structure constants  
2010 Mathematics Subject Classification: 17A32, 17A60

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## The Structure Of Three Dimensional Leibniz Algebras

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In literature, there are many papers on one dimensional and two dimensional Leibniz algebras. The structure of three dimensional Leibniz algebras are more complicated than the structure of one dimensional and two dimensional Leibniz algebras. In this study, our main aim is to investigate three dimensional non-Lie Leibniz algebras and to show that if  $L$  is a three dimensional non-Lie Leibniz algebra, then there exists a one Leibniz algebra which is isomorphic to  $L$ .

Keywords: Lie algebra, Leibniz algebra, structure constants  
2010 Mathematics Subject Classification: 17A32, 17A60

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## Interactive open source e-book: Lectures on Linear Algebra

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The interactive open source e-book presented during this talk is a part of a much bigger project. One of broad aims of this project is to create a generic toolkit that allows building of specific virtual research environments for various purposes: collaborative research, innovative education of mathematics, communication of mathematics. Our e-book will demonstrate some of the features that can be used to enhance learning process.

The e-book presents a collection of lecture notes from a first course on linear algebra which is extended by variety of applications in physics, computer science, economics and many others. Its unique feature lies in interactivity and easiness to use. Along the theory the reader is familiarised with a popular open mathematical software called SageMath. Not only one can see a code or a visualisation, but also program, illustrate and solve oneself a problem that is discussed. This gives an opportunity to experiment and encourages to practice as lengthy computations are left to the computer. The book is self-contained and does not require installing any additional software.

During the talk we will present a few examples of how ready-to-use SageMath can make learning linear algebra accessible and attractive.

**Keywords:** linear algebra, SageMath, mathematics education.

**2010 Mathematics Subject Classification:** 97H60, 15Axx, 9701.

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## Haar Wavelet Technique for Solving Fractional Differential Equations with An Application

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In this article, the numerical solutions of ordinary differential equations of fractional order using Haar wavelet are discussed. Haar wavelet technique is used to approximate the solution of the fractional ordinary differential equation and then, the Haar wavelet method is applied to linearized fractional ordinary differential equations. In each iteration of quasilinearization technique, solution is updated by the Haar wavelet method. The results are compared with the results obtained by the other technique and with exact solutions. Several initial and boundary value problems are solved to show the applicability and efficacy of the Haar wavelet method. An application has been introduced and studied. The numerical results have clearly shown the advantage and the efficiency of the new method in terms of accuracy and computational time.

**Keywords:**Fractional order, Ordinary differential equations, Haar wavelet.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## Climate change impact on Bejaia citrus production from 1983 to 2016

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The objective of this work is to model the production of Bejaia citrus fruits according to the different factors that come from macroeconomic policies, commercial policies, technological changes, climatic hazards ... etc. This is why we used an econometric analysis, of which we divided this paper into two sections, in the first we will build the regression equation of the independent variables by estimating the coefficients of the model by the method. ordinary least squares. The second section, it will be devoted to the tests of meaning of the deferent estimated parameters of the model and the validation of our model, using Eviews software.

The Estimation is given by multiple regression model following formula:

$$LPROD = B_1 * LPRECP + B_2 * LTEMP + B_3 * LSUPER + B_4 * LVEN + B_5 * LHUMD$$

we performed modeling with the logarithms of each series.

PROD: The production of citrus fruits from Bejaia

SUPER: The area: expressed in hectare.

TEMP: Precipitation: represents the total annual rainfall and / or melted snow expressed in millimeters per square meter.

PRECP: Temperature: represent average annual temperatures.

WIND: The wind is the average annual speed expressed in km / h.

HUMID: Moisture: expressed in.

Keywords: multiple linear regression, parametric estimation, statistical tests.

62Jxx: Linear inference, regression.

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## Escape Time Algorithm for Estimating of the Filled Julia Set Fractal Dimension

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The non-geometric and irregular objects are considered as complex patterns. The geometric complexity is measured as space filling capacity by a factor known as a fractal dimension. Different techniques are proposed to find this complexity measure according to the properties of the pattern. This paper is aimed to introduce a method for counting the dimension of the filled Julia fractal set generated by the Escape Time Algorithm using the method of spreading the points inside the proposed window. The resulted dimension is called Escape Time dimension. A new method to compute a correlation dimension of the Filled Julia fractal set is also proposed based on Grassberger-Procaccia algorithm through computing the correlation function. A log-log graph of the correlation function versus the distances between every pair of points in the filled Julia fractal set, is an approximation of the correlation dimension. Finally, a comparison between these two fractal dimensions of the filled Julia fractal set is presented to show the efficiency of the proposed method.

**Keywords:** Fractal dimension, Filled Julia set, Correlation dimension, Escape Time dimension

**2010 Mathematics Subject Classification:** 28A80,37F05, 81Q35.

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## Crossed module aspects of monodromy groupoids

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The notion of monodromy groupoid we consider was introduced by J. Pradines in [4] to generalize the standard construction of a simply connected Lie group from a Lie algebra to a associated construction of a Lie groupoid from a Lie algebroid.

A *group-groupoid* which is quite generally said to be 2-group, is a group object in the category of groupoids or equivalently an internal groupoid in the category of groups.

In this paper we use the techniques of crossed modules to monodromy groupoids for topological group-groupoids and give a direct construction of the monodromy groupoid. This enables us to produce more examples of monodromy groupoids from topological crossed modules.

**Keywords:** Monodromy groupoid, group-groupoid, crossed module.

**2010 Mathematics Subject Classification:** 18D35, 20L05, 22A05

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## Quotients of monodromy groupoids

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The idea of crossed module over groups was initially introduced by Whitehead in [6] during the investigation of the properties of second relative homotopy groups for topological spaces. The categorical equivalence between crossed modules and group-groupoids was proved by Brown and Spencer in [2]. Following this equivalence normal and quotient objects in these two categories have been recently compared and associated objects in the category of group-groupoids have been characterized in [4].

In this paper we use the techniques of crossed modules to obtain the quotient monodromy groupoids together with some properties.

**Keywords:** Monodromy groupoid, quotient groupoid, crossed module.

**2010 Mathematics Subject Classification:** 18D35, 20L05, 22A05

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## Plasma Density Effect on Terahertz Wave Generation due to Interaction of High Intense Laser Field with Magnetized Plasma

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In this article, the governing in initial plasma wave frequency is playing vast role firstly to generate THz coherent wave then to enhance THz wave. Due to relativistic increase of electron mass, the relativistic self-focusing of a RCP laser beam inside magnetized plasma will occur which leading to raise the laser power to enough limits for exciting the terahertz wave. By fulfilling the energy-momentum conservation conditions between the three waves (laser pump wave, plasma wave and terahertz wave), two frequencies (0.5 THz and 5 THz) are obtained. More stabilization and higher power (reaching to tens of gigawatts) of terahertz field amplitude have been observed whenever the plasma frequency are increased. Better results are recording at high THz frequency (5 THz) comparing with low THz frequency (0.5 THz).

**Keywords:** plasma waver, terahertz field, relativistic nonlinearity.

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## Preparation and Characterization of Bimetallic Ag:Cu colloidal Nanoparticles by Pulse Laser ablation PLA for antibacterial application

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**Abstract:** In this paper, Cu and Ag and Bifunctional Cu:Ag colloidal nanoparticles NPs were prepared by using pulse laser ablation in liquid (PLAL) method. Where pure silver and copper plates targets were immersed and ablated in deionized water (DIW) using Nd:YAG Q-Switched Laser with 1064 nm wavelength, 700 mJ energy, 5 Hz frequency, and the number of pulses is 100, 150 and 200 at room temperature. The size distribution, surface plasmon resonance SPR band (absorbance), structure, and surface morphology of the prepared colloidal NPs were investigated by using particle size analyzer, UV-Vis spectrophotometer, X-ray diffraction and transmission electron microscopy. The size distribution results revealed that the average diameters of nanoparticles increase with an increase in the number of laser shots. The SPR band (absorbance) spectra of signifies a single and sharp peaks around (398, 475, and 418) nm for Ag, Cu, Ag:Cu respectively. The results of inhibition zones diameter showed that the NPs have synergistic effects on the studied bacteria up to 15 mm and 4 mm at the highest and lowest values and it increase with the increase of laser pulses number.

**Keywords:** Ag, Cu Colloidal nanoparticles, Bifunctional NPs, Laser ablation .

2010 Mathematics Subject Classification: 03E72, 46S40.

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## On the Krylov subspace method for identifying an unknown source in the heat equation

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In this talk, we are concerned with the problem of approximating a solution of an inverse parabolic problem. In order to overcome the instability of the original problem, we use the truncature spectral method to construct a stable approximate solution. To calculate the stabilized solution, we use a numerical procedure based on the Krylov subspace method. This algorithm provides us a practical and simple method to calculate numerically the stabilized solution. Some Numerical tests are presented to illustrate the accuracy and efficiency of this method.

**Keywords:** Inverse problems, iterative method, Krylov sub-space projection.  
**2010 Mathematics Subject Classification:** Primary 47A52; secondary 65J22.

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## On Classical $n$ -absorbing Submodules

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Let  $R$  be a commutative ring with identity and  $M$  be a unitary  $R$  module. In this paper, we investigate some properties of  $n$ -absorbing submodules of  $M$  as a generalization of 2-absorbing submodules. We also define the classical  $n$ -absorbing submodule, a proper submodule  $N$  of an  $R$ -module  $M$  is called a classical  $n$ -absorbing submodule if whenever  $a_1 a_2 \dots a_{n+1} m \in N$  for  $a_1, a_2, \dots, a_{n+1} \in R$  and  $m \in M$ , there are  $n$  of  $a_i$ 's whose product with  $m$  is in  $N$ . Furthermore, we give some characterizations of  $n$ -absorbing and classical  $n$ -absorbing submodules under some conditions.

**Keywords:**  $n$ -absorbing submodule, Classical  $n$ -absorbing submodule.

**2010 Mathematics Subject Classification:** 13C05, 13C13, 13C99.

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## Global Convergence of Conjugate Gradient Method in Unconstrained Optimization Problems

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In this paper, we suggest a new parameter in conjugate gradient method . The new method fulfils the sufficient descent condition with the strong Wolfe condition when inexact line search is used. The numerical results of the suggested method shows this method outperforms to other standard conjugate gradient method.

Keywords Unconstrained optimization problem Conjugate Gradient Method , Inexact line search.  
2010 Mathematics Subject Classification: 03E72, 46S40.

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## A new Conjugacy Coefficient of Conjugate Direction Method in Unconstrained Optimization

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**Abstract** We proposed a new conjugate coefficient constructed, which is using the notion of conjugate direction, it satisfies the descent condition and under some conditions the global convergence is proved. The numerical results show that our method is very efficient when compare to other classical method.

**Keywords:** Robot, simultaneous of optimal control, non-classical variational approach .

**2010 Mathematics Subject Classification:** 70E60, 49N05,70B15

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## A method for estimating RCA models

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In this work, we are interested in the statistical inference of a class of first order Random Coefficient Autoregressive models (RCA). After describing the RCA model and reviewing its probabilistic properties and interactions with some time series models, we presented a four-stage weighted least squares (4S-WLS) estimator and its asymptotic properties independently of the condition of stationarity [1]. We have performed a finite sample comparison of this estimator with the quasi-maximum likelihood estimator (QMLE)[2]. The asymptotic results obtained were illustrated by simulation studies which have shown that our method better estimates the parameters of the model even outside the domain of stationary. In addition, this estimator has the same asymptotic variance as the Gaussian QMLE independently of the stability hypothesis.

**Keywords:** RCA models, strict stationarity, four-stage weighted least squares estimator, quasi-maximum likelihood estimator, asymptotic normality.

**2010 Mathematics Subject Classification:** 62M10, 62M04.

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## Riesz transforms for the Weinstein operator

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In this paper we study the Riesz transforms  $\mathcal{R}_w$  related to the Weinstein operators  $\Delta_w = \sum_{i=1}^d x_d^{-2\alpha-1} \frac{\partial}{\partial x_i} \left( x_d^{2\alpha+1} \frac{\partial}{\partial x_i} \right)$ . We develop for  $\mathcal{R}_w$  a theory that runs parallel to the one for the Euclidean Riesz Transform. It is proved that the Riesz-Weinstein transform in coordinates  $i = 1, \dots, d$ ,  $\mathcal{R}_w^i$  is actually a Calderón-Zygmund singular integral operator in the sense of the associated space of homogeneous type. Moreover, our Riesz-Weinstein transform can be written as a principal value.

**Theorem 1.** Assume that  $\alpha \geq -\frac{1}{2}$ . Then the Riesz-Weinstein transforms  $\mathcal{R}_w^j$ ,  $j = 1, \dots, d$ , can be uniquely extends to  $L^p(\mathbb{R}_+^d, \Phi d\mu_\alpha)$  as a bounded operators from  $L^p(\mathbb{R}_+^d, \Phi d\mu_\alpha)$  into itself,  $1 < p < \infty$ ,  $\Phi \in A_p^\alpha$ , and to bounded operators from  $L^1(\mathbb{R}_+^d, \Phi d\mu_\alpha)$  to  $L^{1,\infty}(\mathbb{R}_+^d, \Phi d\mu_\alpha)$ ,  $\Phi \in A_1^\alpha$ .

**Keywords:** Weinstein's operator, Riesz transforms, Spaces of homogeneous type.

**2010 Mathematics Subject Classification:** 26A33 ,42B10, 42B20.

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## Deriving Cramer-Rao Bounds of Frequency Estimator for Signal with Multiplicative Noise

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The work in this paper produce mathematically deriving for Cramer-Rao bound (CRB)expression of frequency estimator for single tone sinusoidal signal under zero mean Gaussian multiplicative noise and additive Gaussian noise (AWGN). Probability density function of proposed signal model are derived. Mathematical and comparative study support the obtained simulation results

Keywords: Multiplicative noise, frequency estimator, CRB  
2010 Mathematics Subject Classification: 03E72, 46S40.

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## Binary Operation Graphs

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In this paper, a new concept of graph labeling called binary operation labeling is introduced. Let  $G = (V, E)$  be a  $(n, m)$ -graph and let  $f: V(G) \rightarrow \{1, 2, \dots, n\}$  be a bijection. We define  $f^*$  on  $E(G)$  by  $f^*(uv) = (f(u) + f(v))/2$  if  $f(u)$  and  $f(v)$  both are odd or both are even and  $f^*(uv) = (f(u)f(v))/2$  if  $u$  is odd and  $v$  is even or vice versa for each  $uv \in E(G)$ . If  $f^*$  is injective on  $E(G)$ , then  $f$  is called a binary operation labeling. The graph  $G$  is said to be a binary operation graph if  $G$  admits a binary operation labeling. In this paper some results for this new type of labeling are introduced.

**Keywords:** binary operation graphs, maximal binary operation graphs  
**2010 Mathematics Subject Classification:** 05C69, 05C78, 97K30.

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## Comparative study of four predictive equations of basic metabolism

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Several predictive equations of the basic metabolism (MB) have been developed and allow us an estimation of the energy needs of the body. The purpose of this study is the Analysis and comparison of four MB estimation equations. For these, 241 participants aged between 19 and 22 years participated in this study. For each participant, the MB expressed in calories is estimated by the following equations: Harris and Benedict; Roza; WHO and Black. The comparative study was conducted for both sexes in the total population but also in relation to their body mass index (BMI). The different comparisons are made by Wilcoxon's signed rank test. The obtained results show that in women, the comparative study between the four equations in the total population, the one with lean BMI and the one with normal BMI showed statistically significant differences (p-value inferior than 0.05). In women with high BMI, the comparison showed that only Haris-Black, Haris-WHO and Roza-WHO are statistically no different (p-value superior than 0.05). In men, this comparison made among the population with Normal BMI showed statistically significant differences (p-value inferior than 0.05). Only comparisons between Haris-Roza and Roza-Black in the total population as well as Haris-WHO and Roza-WHO in the high-BMI population do not show statistically significant differences (p-value superior than 0.05). In conclusion, the prediction of basal metabolism by these four equations showed statistically significant differences in the total population but also in body composition.

**Keywords:** Predictive equation, Basic metabolism, Wilcoxon test.

## Theoretical study of ThGeO<sub>4</sub> compounds as a new host material

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In order to find ways to understand the origin of bonding, electronic and thermodynamic properties of the of the ThGeO<sub>4</sub> host material [1]. We report here first principles calculations within the density functional theory framework based pseudo-potential method. Our calculations shows that the Zircon-type ThGeO<sub>4</sub> undergoes a phase transition to tetragonal scheelite structure, and beyond to monoclinic fergusonite ones [2]. Attention was placed on the trends of the dynamical stability and anisotropic behavior related to structural phase transition. Linear as well as cubic thermal expansion component yields dierent directional dependence as a function of temperature for the investigated polymorphs of the compound. We attribute the origin of the difference of expansion in the unit cell to the distortion of the ThO<sub>8</sub> dodecahedra. Additionally, analysis of the non-covalent dispersion of the zircon and scheelite structures reveals a counter-balance between the destabilizing interactions (due to steric crowding) and the current attractive and repulsive ones.

**Keywords:** Ab initio study, host material, charge density .  
**2010 Mathematics Subject Classification:** 82D25.

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## Integral type contractions in partial metric spaces

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Let  $X$  be a nonempty set and  $f : X \rightarrow X$  be a mapping. If  $f(x) = x$ , for some  $x \in X$ , then  $x$  is fixed point of  $f$ . Banach fixed point theorem was introduced in 1922 in complete metric spaces as “ $(X, d)$  be a complete metric space and  $f : X \rightarrow X$  be a selfmapping. If there exists  $0 \leq k < 1$  such that  $d(fx, fy) \leq kd(x, y)$  for all  $x, y \in X$ . Then  $f$  has unique fixed point”.

Partial metric spaces were introduced by Matthews (1994) as a generalisation of usual metric spaces where the self distance for any point need not be equal to zero. In this work, we defined generalized integral type contractions using altering distance functions and proved common fixed point theorems for four mappings satisfying this type (Branciari type) contractions in partial metric spaces.

**Keywords:** contraction, fixed point, partial metric.

**2010 Mathematics Subject Classification:** 54H25, 49H10.

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## Difference scheme for the parabolic equation

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### Abstract:

In this study, the first order of approximation of the difference scheme for the approximate solution of the parabolic equation  $u_t + Au = f(t), t > 0$  in a Hilbert space  $H$  with self-adjoint operator  $A$  is presented. The well posedness of this difference scheme is established. The numerical solution is given.

**Key words:** Difference scheme, Parabolic equation, Stability, Well posedness.

2010 Mathematics Subject Classification: 46E20.

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## On Harmonious Labelling of Triangular Bridge

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This paper discusses about one of the labeling on the graph that is harmonious labeling. Graph that has harmonious labeling is called harmonious graph. Examples of graphs that have harmonious labeling described in this article are path, ladder, triangular ladder, triangular ladder pendant and triangular fence. The main result in this paper is to show that triangular bridge is one of harmonious graph.

**Keywords:** harmonious labelling, triangular bridge, ladder.

**2010 Mathematics Subject Classification:** 05C78.

**Acknowledgment:** This work was supported by Indonesian Endowment Fund for Education (LPDP Indonesia).

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## The $k$ -metric dimension of double fan graph

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Given a connected simple graph  $G = (V, E)$  and a positive integer  $k$ . A set  $S \subseteq V$  is said to be a  $k$ -metric generator if and only if for any pair of different vertices  $u, v \in V$ , there exist at least  $k$  vertices  $w_1, w_2, \dots, w_k \in S$  such that  $d(u, w_i) \neq d(v, w_i)$ , for every  $i \in \{1, 2, \dots, k\}$ , where  $d(u, v)$  is the length of a shortest  $u - v$  path. A  $k$ -metric generator of minimum cardinality is called  $k$ -metric basis and its cardinality, the  $k$ -metric dimension of  $G$ . We define double fan graph is join between  $N_2 + P_n$ . In this article we determine the  $k$ -metric dimension of double fan graph.

**Keywords:**  $k$ -metric generator,  $k$ -metric dimensional,  $k$ -metric dimension,  $k$ -metric basis, double fan graph.

**2010 Mathematics Subject Classification:** 05C12

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## Stability Inequalities in a Periodic Inventory Model with $(R, s, lQ)$ Policy

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In this work, we consider the single-item and single-step periodic inventory model with  $(R, s, lQ)$  policy. Firstly, we determine the transition kernel of the Markov chain describing our system. Secondly, we establish the strong V-Stability results of the studied system in order to determine the stability inequalities to estimate the committed error. Finally, a numerical application will be designed to valorize the obtained results.

Keywords: Inventory models, Markov Chains, Modeling, Approximation, Stability Estimates.  
 2010 Mathematics Subject Classification: 36A18, 30E05, 11K50.

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## Baysian Inference of the Reliability Function for Gompertz Distribution

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In this paper, some Bayes estimators of the reliability function of Basic Gompertz distribution have been derived based on generalized weighted loss function. In order to get a best understanding of the behavior of Bayesian estimators, a non-informative prior as well as an informative prior represented by exponential distribution is considered. Monte-Carlo simulation have been employed to compare the performance of different estimates for the reliability function of Basic Gompertz distribution based on Integrated mean squared errors. Finally, a discussion is provided to illustrate the results that are summarized in tables.

**Keywords:**Gompertz distribution, reliability function; Bayes estimator;

**2010 Mathematics Subject Classification:** 03E72, 46S40.

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## Two Impartial Games on Finite $p$ - Groups

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This paper is about two impartial combinatorial games, namely achievement and avoidance games. These games can be played on groups by two persons. If  $G$  is a finite group, then both players chose elements of  $G$  alternately. The player who builds a generating set for the group  $G$  is called the loser in avoidance game whereas the builder of generating set is called the winner in achievement game. Since, the nim-numbers provide a more easier analysis of impartial combinatorial games and provide final outcome of these impartial games. But it is hard to calculate the nim-numbers of an impartial game. In this paper, the nim-numbers for achievement and avoidance games on some non-abelian groups have been calculated. In particular, the simplified structure diagrams and nim-values of these two games for finite non-abelian  $p$ -groups, dicyclic type groups, and groups of the form  $S_3 \times C_p$  have been determined.

**Keywords:** Nim-value, structure class, simplified diagrams,  $p$ - groups.

**2010 Mathematics Subject Classification:** 91A46, 20D30.

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## Synthesis and Characterization of Conducting Polymers and Soluble Application in Organic Photovoltaic Cells

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In this work we have shown that the new polymerization process catalyzed by Mag-H<sup>+</sup> can be applied to the polymerization of the pyrrole derivatives with nitrobenzaldehyde. Was synthesized poly (2,5-diyl 2,4-dinitrobenzylidene pyrrole), poly (2,5-pyrrole -4-nitrobenzaldehyde) and poly (2,5-pyrrole-3- nitrobenzaldehyde) with Mag-H<sup>+</sup>. The polymerization is done in solution, heterogeneous catalyst. The return on the amount of Mag-H<sup>+</sup> 10% compared to the mass [Pyrrole] / [Benzaldehyde] = 50/50 is present a good yield. The data of different analytical methods used in this study: UV-visible and IR allowed to prove the characteristic bands of the synthesized polymers. The synthesized polymers were deposited on glass substrates and characterized by electron microscopy Atomic Force Microscopy (AFM), scanning electron microscopy (SEM) and optical measurements (transmission, absorption). The optical measurements of PPDNB show that it is a semiconductor. The PPDNB, PPNB (4), PPNB (3) obtained are soluble in the organic solvent, chloroform, toluene, NMP, acetonitrile, acetone and insoluble in water and methanol.

**Keywords:** Conducting Polymer, Organic Solar Cells, Spin-Coating, Mag-H<sup>+</sup>.

**2010 Mathematics Subject Classification:** 78A55.



## Detection for the clotting in the vascular network by geometric combinatorics

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Humans have suffered a lot of side effects when doing a scan either by CT phrases, magnetic resonance and rays. In this work, different way was used to reduce these affects depends on segmentation of vascular networks then represents it as a graph which made each intersection as a vertex and the line between them represents the edge. Now we want to find the place of the clot. This can be done by represents the weight of each edge as the amount of the blood in the vessel. If the amount is less than the normal flux then we put a sign which represents the place of thrombosis. If more than one thrombosis exists then algorithms in graph theory are used to find the minimum distance to reach the nearest one and sign it alternately.

**Keywords:** Vascular network, connected graph, algorithm.

**2010 Mathematics Subject Classification:** 92C42, 05C40, 05C85.

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## Some New Conformable Fractional Integral Inequalities for Symmetrized Convex Functions

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Since the so-called Hermite-Hadamard type inequalities for convex functions were presented, their generalizations, refinements, and variants involving various integral operators have been extensively investigated. Here we aim to establish several Hermite-Hadamard inequalities and Hermite-Hadamard-Fejér type inequalities for symmetrized convex functions and Wright-quasi-convex functions with a weighted function symmetric with respect to the midpoint axis on the interval involving the fractional conformable integral operators initiated by Jarad [4] We also point out that certain known inequalities are particular cases of the results presented here.

**Keywords:** Symmetrized convex function, Hermite-Hadamard type inequality, Hermite-Hadamard-Fejér type inequality.

**2010 Mathematics Subject Classification:** 26A33, 26D10, 26D15, 33B20.

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## On the product of Borel and logarithmic methods of summability

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In this study, some two-sided Tauberian conditions are obtained for the product of Borel and logarithmic methods of summability.

**Keywords:** Tauberian theorem, Borel summability, logarithmic summability.

**2010 Mathematics Subject Classification:** 40E05, 40G10, 40C05.

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## Theorems on the subsequential convergence

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Subsequential convergence of a real sequence was defined by Stanojević [1] as follows:

**Definition 1.** If there exists a finite interval  $I$  such that all accumulation points of a sequence  $\{s_n\}$  are in  $I$  and every point of  $I$  is an accumulation point of  $\{s_n\}$ , then  $\{s_n\}$  is called subsequentially convergent.

All convergent sequences are subsequentially convergent. Further, it is clear from the definition that subsequential convergence implies boundedness. However the converse is not necessarily true, provided by the example  $\{(-1)^n\}$ . The first theorem which reveals that the converse is true under several condition was given by Dik [2].

In this paper we introduce new conditions under which subsequential convergence of a real sequence follows from its boundedness.

**Keywords:** Subsequential convergence, slowly oscillating sequences.

**2010 Mathematics Subject Classification:** 40A05, 40E05.

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## Model Order Reduction for Greenhouse Cayenne Pepper Crop Production Model

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In this paper, it will discussed the dynamic model of the greenhouse cayenne pepper crop production. The cayenne pepper crop is one of plants which susceptible to the micro climate alteration. The dynamic model of greenhouse cayenne pepper crop production have a high order which it makes difficult in analyze the model and design a controller. Therefore, model order reduction is done by the balanced truncation method. The properties of the reduced linear model is similar with the original model.

**Keywords:** model order reduction, greenhouse, balanced truncation.

**2010 Mathematics Subject Classification:** 93B20, 37C00, 93C04.

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## On the stability of two functional equations arising in mathematical biology and theory of learning

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In this study, we prove the Hyers-Ulam stability and the Hyers-Ulam-Rassias stability of the following two functional equations

$$\varphi(x) = x\varphi((1 - \alpha)x + \alpha) + (1 - x)\varphi((1 - \beta)x), \quad x \in [0, 1], \quad 0 < \alpha \leq \beta < 1,$$

and

$$\varphi(x) = x\varphi(f(x)) + (1 - x)\varphi(g(x)), \quad x \in [0, 1]$$

which is an open problem raised by Berinde and Khan [1].

**Keywords:** Functional equation, Hyers-Ulam stability, Hyers-Ulam-Rassias stability.

**2010 Mathematics Subject Classification:** 34K20, 39B05, 47H10.

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## A brief overview of the encryption methods used throughout the history of cryptography

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Cryptology is a cryptography science based on mathematical. Throughout history, people needed some kind of privacy in communication. As early as 1900 B.C., Egyptian scribes used hieroglyphs in a non-standard fashion, presumably to hide the meaning from those who did not know the meaning [1]. Cryptography is the effect of designing privacy requirements systems in communication [2]. In the past, many classical encryption methods such as Caesar, Vigenere, Enigma have been used. Today, more powerful modern encryption methods such as RSA, EEC, SHA256, MD5 are used.

This paper aims to present a brief history of cryptography.

**Keywords:** History of cryptology, cryptography, cryptanalysis.

**2010 Mathematics Subject Classification:** 97R40, 94A60, 11T71.

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## A study of cryptology and new generation keys to secure the security of information between objects

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In the past decade, internet of things (IoT) has been a focus of research. Security and privacy are the key issues for IoT applications, and still face some enormous challenges [1]. Many innovations such as Internet of things (IoT), machine-to-machine communication (M2M), artificial intelligence and block chain technology of objects, and the existing classical solutions for the diversity and increase of cyber security in their applications are insufficient. Cyberattacks on IoT systems are very critical since they may cause physical damage and even threaten human lives [2]. The change in technology, practice and individual expectations also necessitates a change in security solutions and techniques. Security in networking is based on Cryptography the science [3]. Cryptology is a mathematical subclass of cryptography. Key methods used in this area are a change over time. Each new method takes over the flag from the previous method. This search for solutions continues uninterruptedly.

This paper presents a new perspective on key next-generation key solutions from classical approaches in providing inter-object security.

**Keywords:** Internet of things (IoT), information security, cyber-attack, cryptography, block chain.  
**2010 Mathematics Subject Classification:** 11T71, 14G50, 94A60.

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## Lacunary $A$ –statistical convergence and lacunary strong $A$ –convergence of order $\alpha$ with respect to a modulus

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In this paper, the definitions of lacunary strong  $A$ –convergence of order  $\alpha$  with respect to a modulus and lacunary  $A$ –statistical convergence of order  $\alpha$  were given by adding  $\alpha$  to the definitions of lacunary strong  $A$ –convergence with respect to a modulus and lacunary  $A$ –statistical convergence when  $A = (a_{ik})$  is an infinite matrix of complex numbers and  $0 < \alpha \leq 1$ . We studied some connections between lacunary strong  $A$ –convergence of order  $\alpha$  with respect to a modulus and lacunary  $A$ –statistical convergence of order  $\alpha$ .

**Keywords:** Statistical convergence, modulus function, lacunary sequence.

**2010 Mathematics Subject Classification:** 40A05, 40C05, 46A45.

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## On $I$ – deferred statistical convergence of order $\alpha$

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The idea of  $I$ –convergence of real sequences was introduced by Kostyrko et al. [ *Kostyrko, P., Šalát, T. and Wilczyński, W.  $I$ –convergence, Real Anal. Exchange 26(2) (2000/2001), 669–686* ] and also independently by Nuray and Ruckle [ *Nuray, F. and Ruckle, W. H. Generalized statistical convergence and convergence free spaces. J. Math. Anal. Appl. 245(2) (2000), 513–527* ]. In this paper we introduce  $I$ –deferred statistical convergence of order  $\alpha$  and strong  $I$ –deferred Cesàro convergence of order  $\alpha$  and investigate between their relationship.

Keywords: Statistical convergence,  $I$ –convergence, Cesàro summability.

2010 Mathematics Subject Classification: 40A05, 40C05, 46A45.

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## A variation on $N_{\alpha}^{\beta}(\theta, I)$ – ward Continuity

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The main purpose of this paper is to introduce the concept of strongly ideal lacunary quasi-Cauchyness of order  $(\alpha, \beta)$  of sequences of real numbers. Strongly ideal lacunary ward continuity of order  $(\alpha, \beta)$  is also investigated. Interesting results are obtained.

**Keywords:** Lacunary sequence, summability, continuity.

**2010 Mathematics Subject Classification:** 40A05, 46B20, 47H09, 47H10.

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## Investigation of the Normal Stress In An Infinite Elastic Body With A Locally Curved Carbon Nanotube

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In this study, the results of normal stress values in unidirectional fibrous composite with locally curved carbon nanotube (CNT) were obtained as much as the second approximation and the obtained results were analyzed. For these investigations, were used in the framework of the piecewise homogeneous body model the three-dimensional geometrically nonlinear exact equations of elasticity theory. The boundary value problem is solved by using boundary form perturbation method. For simplicity, a small carbon nanotubes is considered and interaction between the carbon nanotubes is not into account.

**Keywords:** Carbon Nanotube, stress distribution, locally curved.

**2010 Mathematics Subject Classification:** 74E05,74D10 , 74E10.

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## Examination Of The Self- Balanced Normal Stress In An Elastic Body With A Single Locally Curved And Hollow Nanofiber

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In this study, the self-balanced normal stress distribution in an infinite elastic body with a locally curved and hollow nanofiber was investigated and the corresponding stress values were obtained as much as the second approximation. The investigations were carried out by using of three-dimensional exact equations of elasticity theory in a piecewise homogeneous body model framework. It assumed that a single locally curved and hollow nanofiber of infinite length is located in an infinite elastic body low concentration nanofibers. The interaction between the nanofibers is neglected. Numerous results of self-balanced normal stresses were presented and analyzed.

**Keywords:** Self balanced stress, stress distribution, nanofiber.

**2010 Mathematics Subject Classification:** 74B15,74D10 , 74E10.

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## Numerical analysis of star network

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The main purpose of the current paper is to use Taylor series expansion approach for analyzing of the star network, with an emphasis on parametric uncertainty analysis. Additionally, we approximate the expectation and the variance of the performance measures and we compare it to the corresponding Monte Carlo simulation results.

### Approach development

In this work, we consider a star network [1] where we assume that parameter values of model have uncertainty. We use the Taylor series expansions approach to estimate the expectation and variance of model output, due to parametric uncertainties in the model input parameters [2].

**Keywords:** A star network, Taylor series expansion, Monte Carlo simulation.

**2010 Mathematics Subject Classification:** 65D15, 60J10 , 65C05.

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## A study on Absolute Almost Convergence

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In this paper, we give a study of  $(P_n, s)$ -absolutely almost convergence where  $np_n = o(P_n)$  as  $n \rightarrow \infty$ ,  $P_n = \sum_{i=0}^n p_i$ , and  $s > 0$ .

**Keywords:** Sequences, series, absolutely almost convergence.

**2010 Mathematics Subject Classification:** 40D05, 40D15, 40F05.

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## Effects of Some Graph Operations on Total Influence Number

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The parameter called total influence number of a graph, denoted by  $\eta_T(G)$ , was introduced by Daugherty and et al. This is the extension of the graph parameter known as the influence number,  $\eta(G)$ . In this paper, total graphs and extended fibonacci cubes are taken attention. Also, some results about total influence number of extended fibonacci cubes and total graph of some special graphs are obtained.

**Keywords:** Graph theory, total influence number, graph operations.

**2010 Mathematics Subject Classification:** 05C99, 05C76

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## $\beta_1$ –paracompactness in ideal topological spaces

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In this paper, the class of  $\beta_1$ – $\mathcal{I}$ –paracompactness in ideal topological spaces is introduced. Then, some fundamental properties of the space and its relation between some other types of paracompact spaces are investigated. Also some counterexamples concerning the space are given.

**Keywords:** Paracompact,  $\beta$ –open set,  $\beta_1$  –  $\mathcal{I}$ –paracompact.

**2010 Mathematics Subject Classification:** 54D20, 54G20, 54A10.

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## Effect of some climate attributes on rice productivity in yogyakarta province, indonesia using vector autoregression model

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Unpredictable weather is one of the main causes of declining rice yields. As already known that the weather at a location can be different from other locations. This difference leads to the possibility that the risk of harvest failure at a site will be different from that of the other location. Therefore, in this research will be studied how the impact of weather in agriculture especially rice farming in Yogyakarta Province, Indonesia. First, the study identify attributes of climate that significantly influence rice productivity. Second, it uses vector autoregressive model to estimate their impact on rice productivity and forecasting in the future. Time series data of rice productivity and some attributes of climate are used in this work.

**Keywords:** Forecasting, rice productivity, vector autoregressive model.

**2010 Mathematics Subject Classification:** 62G25, 62M10, 62M20.

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## Fréchet-Hilbert spaces and the SCBS property

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In this paper we study Fréchet-Hilbert spaces to investigate isomorphic classification of Cartesian products of Fréchet spaces. We obtain that all separable Fréchet-Hilbert spaces have the smallness up to a complemented Banach subspace (SCBS) property. For that reason, we can show that the bounded perturbation of an automorphism on a separable Fréchet-Hilbert space still takes place up to a complemented Hilbert subspace. In addition, the strong dual of Fréchet-Hilbert spaces also satisfies the SCBS property.

**Keywords:** Fréchet-Hilbert spaces, the SCBS property, bounded operators.

**2010 Mathematics Subject Classification:** 46A03, 46A45, 47B37.

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## Reduction method for second order state dependent sweeping process

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In this paper, we present a new approach for solving second order nonconvex perturbed sweeping process in finite dimensional setting. It consists in a reduction of the problem to a first order one without use of the standard methods of fixed point theory. The perturbation, that is the external force applied on the system, is not necessary with bounded values. Further, in infinite Hilbert space, we provide an existence result when the perturbation contains a finite delay.

**Keywords:** state-dependent sweeping process, prox regular sets, unbounded perturbation, reduction.  
**2010 Mathematics Subject Classification:** 34A60, 49J52, 49J53.

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## A fast relaxed normal two split method and an effective weighted TV approach for Euler's elastica image inpainting

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In this talk I propose two numerical algorithms for solving Euler's elastica-based inpainting model. The minimizing functional is non-smooth, non-convex, and involves high-order derivatives, that traditional gradient descent based methods converge very slowly. Recent alternating minimization methods show fast convergence when a good choice of parameters is used. The objective is to introduce efficient algorithms which have simple structures with a fewer parameters. These methods are based on operator splitting and alternating direction method of multipliers, and subproblems can be solved efficiently by Fourier transforms and shrinkage operators. For the first method, we relax the normal vector in the curvature term of the Euler's elastica model and exploit two operator splitting techniques to propose a Relaxed Normal two Split (RN2Split) method. The second method,  $\kappa$ -weighted Total Variation ( $\kappa$ TV), solves the Euler's elastica minimization problem as a weighted total variation. Some analytical properties of each algorithm will be presented. Various numerical experiments, including comparison with some existing state-of-art algorithms, are presented to show the efficiency and the effectiveness of the proposed RN2Split and  $\kappa$ TV methods.

Keywords: Euler's elastica, optimization, alternating minimization, fast methods, image inpainting  
2010 Mathematics Subject Classification: 94A08, 49-XX, 90C26.

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## Soft $D$ -Metric Spaces

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The first aim of this paper is to define soft  $D$ - metric spaces and to give some fundamental definitions. In addition to this, by using the concept of soft  $D$ - metric, we define a soft  $\Delta$ - distance on a complete soft  $D$ - metric. Finally, we give the fixed point theorem of soft continuous mappings on soft  $D$ - metric spaces.

**Keywords:** Soft set, generalized soft  $D$ - metric space, soft contractive mapping, fixed point theorem.  
**2010 Mathematics Subject Classification:** 54H25, 47H10, 54E99.

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## Variations on lacunary statistical quasi Cauchy sequences

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A sequence  $(x_k)$  is lacunary statistically  $p$ -quasi-Cauchy if  $\lim_{r \rightarrow \infty} \frac{1}{h_r} |\{k \in I_r : |x_{k+p} - x_k| \geq \varepsilon\}| = 0$  for each  $\varepsilon > 0$ . A function  $f$  is called lacunary statistically  $p$ -ward continuous on a subset  $A$  of the set of real numbers  $\mathbb{R}$  if it preserves lacunary statistically  $p$ -quasi-Cauchy sequences. We prove compactness and continuity type theorems.

Keywords: Summability, lacunary statistical convergence, continuity.  
 2010 Mathematics Subject Classification: 40D15, 42A55, 46H40.

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## A new result on weighted arithmetic mean summability of almost increasing sequences

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In this work, a known theorem of Bor concerning about weighted mean summability of non-decreasing sequences has been generalized to  $|A, p_n; \delta|_k$  summability factors of almost increasing sequences. Also some new results have been obtained concerning  $|\bar{N}, p_n|_k$ ,  $|\bar{N}, p_n; \delta|_k$  and  $|C, 1; \delta|_k$  summability factors.

**Keywords:** Absolute matrix summability, Fourier series, infinite series.

**2010 Mathematics Subject Classification:** 40A05, 40D25.

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## A new note on absolute summability factors of Fourier series

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Quite recently, Bor in [Anal. Math. Phys., An application of quasi-monotone sequences to infinite series and Fourier series, 8,(2018), no.1, 77-83] obtained two main theorems concerning the weighted mean summability factors of infinite series and trigonometric Fourier series and generalized main theorems under more weaker conditions. In this present paper, we extended these main theorems to absolute matrix summability method by using normal matrices instead of weighted mean matrices.

**Keywords:** Summability factors, absolute matrix summability, Fourier series, infinite series.

**2010 Mathematics Subject Classification:** 40A05, 40D25, 26A15.

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## Complete lift of tensor fields of type $(1,0)$ on cross sections in semi-cotangent bundles

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The aim of this work is to investigate cross-sections in semi-cotangent (pull-back) bundle  $t^*M$  of cotangent bundle  $T^*M$  by using projection (submersion) of the tangent bundle  $TM$ .

**Keywords:** Vector field, pull-back bundle, cross-section, semi-cotangent bundle.

**2010 Mathematics Subject Classification:** 53A45, 53C55, 55R10.

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## Ternary relations: An overview of theory and applications

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In recent years, the interest in ternary relations is on the rise. They appear in some area of pure and applied mathematics (e.g. median algebra and group theory [3, 4], order theory [5] and formal concept analysis [2]). Also, they have become essential tools in various fields of applied sciences, including, computer sciences [1], biology [7]), and social sciences [6].

In this talk, we aim to present an overview of theory and applications of ternary relation [8]. Also, we extend some interesting notions and properties of binary relations to the setting of ternary relations.

**Keywords:** Ternary relation, Traces, Relational compositions.

**2010 Mathematics Subject Classification:** 03E20, 03D20, 03E72.

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## Ideals and filters on a trellis

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The notion of ideal or its dual (filter) recognized as one of the most important concepts in lattice theory [1, 3], and theory of other algebraic structures used in logic (see, e.g., [2, 6, 7]). In this talk, we present an extension of the notion and properties of ideal (resp. filter) on a lattice without the assumption of transitivity. For this purpose, we use the notion of trellis [4, 5] which is considered as an extension of the notion of lattice by only eliminate the property of transitivity.

**Keywords:** Ideal, Filter, Pseudo ordered set, Trellis.

**2010 Mathematics Subject Classification:** 03G10, 06B10.

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## Recent Results for the Logarithmic Keller-Segel-Fisher/KPP System

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We consider a Keller-Segel type chemotaxis model with logarithmic sensitivity and logistic growth. It is a  $2 \times 2$  system of partial differential equations, describing the interaction of cells and a chemical signal. The logarithmic singularity can be removed via the inverse Hopf-Cole transformation. The resulting system is one of hyperbolic-parabolic balance laws, and fits in a general framework proposed and studied by the first author in a sequence of recent papers. In particular, for Cauchy problem with Cauchy data being small perturbations of a constant equilibrium state, we have existence of solution global in time,  $L^2$ -decay of solution to the constant equilibrium state, and a predetermined asymptotic solution in the  $L^2$  sense.

We continue to study the Green's function of the corresponding linear system, linearized around the constant equilibrium state. The result is a detailed, pointwise description of the Green's function: The leading term is a heat kernel. If at least one of the two diffusion coefficients is zero, (say, if the chemical signal is non-diffusive), the Green's function contains  $\delta$ -functions as well. The remainder is a higher order term of the heat kernel, with a faster time decay rate by  $(t + 1)^{-1/2}$ . Our work is a part of the effort to obtain more detailed information of the convergence of the solution to the nonlinear chemotaxis model to its asymptotic solution via Duhamel's principle. In addition, the Green's function has its own significance as the fundamental solution of the linear problem.

**Keywords:** hyperbolic-parabolic, fundamental solution, chemotaxis.

**2010 Mathematics Subject Classification:** 35A08, 35M31, 35Q92.

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## Short–time stability of the solar system

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Consider the Sun, Mercury, Venus, Earth+Moon, Mars, Jupiter, Saturn, Uranus and Neptune as point masses, moving according Newton’s inverse–square law of gravitation. We prove that the present configuration will be stable at least during the next 100 000 years, in sense that any semi–major axis would not change significantly and every eccentricity and inclination will remain bounded.

Keywords: Solar system, stability.

2010 Mathematics Subject Classification: 70F15.

## Analysis of a batch arrival retrial queue with server subject to breakdowns and repairs

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The aim of this work is to present some numerical examples for the model of batch arrival unreliable server subject to breakdowns and repairs, where server is subject to breakdowns and repairs. The customer, whose service is interrupted, stays in the service, waiting for repair of the server. Moreover, after repairing this customer completes his service. By using supplementary variables technique and probability generating function method, we found a stability condition for this system. In the steady state, the explicit expressions for joint distribution of the server state and queue size distribution at various epochs are obtained, full measures of the system, such as the probability of an empty system, the mean number of customers in the retrial queue and the expected retrial time. Finally, the sensitivity of different parameters on various perform has been examined numerically by taking an illustration.

**Keywords:** retrial queue, batch arrival, breakdowns, Repairs, supplementary variables technique.

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## The existence of a weighted mean for Stepanov almost periodic functions

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In this paper, we present some sufficient conditions which do guarantee the existence of a weighted mean for Stepanov almost periodic functions, which will then coincide with the classical Bohr mean. Moreover, we will show that under those conditions, the corresponding weighted Bohr transform exists.

**Keywords:** Stepanov almost periodic functions; weighted mean; Bohr spectrum.  
**2010 Mathematics Subject Classification:** 34G20, 35B10, 37B55, 47D06.

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